Zearn Math Curriculum Study Professional Development Final Report

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Executive Summary

Zearn developed Curriculum Study Professional Development (CS PD) to support elementary school teachers who implement Zearn Math in gaining pedagogical content knowledge (PCK). Research has identified PCK as an important aspect of developing teachers' capacity to teach mathematics. The concept of PCK, as initially defined by Lee Shulman (1986), includes both subject-matter-specific knowledge and subject-matter-specific pedagogy. Later scholars further differentiate the special knowledge that teachers have, including the category of understanding student thinking specific to school content.

SRI Education and TERC, funded by the Carnegie Corporation of New York and the Charles and Lynn Schusterman Family Foundation, developed a multiple-case study to investigate whether and how participation in CS PD led to growth in PCK.

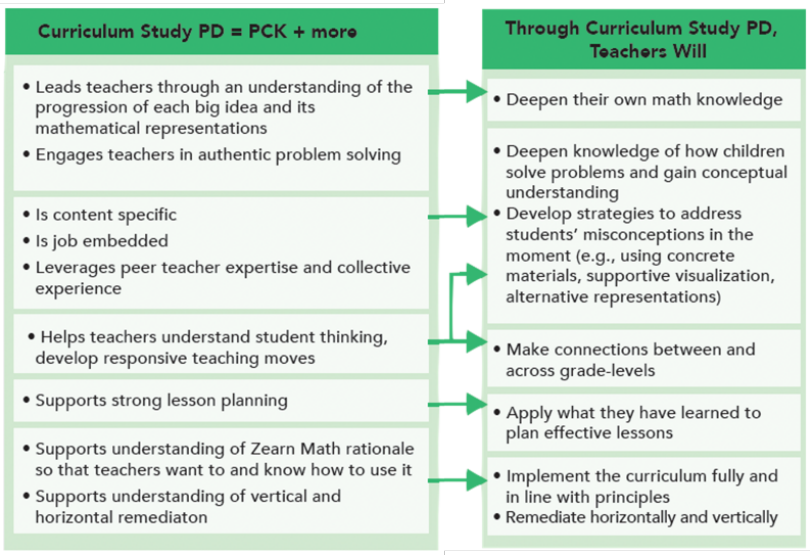
Zearn Math

Zearn Math combines hands-on teaching and immersive digital learning. Its interactive digital lessons include dynamic visual models to help students make sense of math, real-time feedback on problems, and integrated student supports. The Zearn Math approach to fractions—the focus of this study—is to lead students through a series of representations of fractions: concrete, pictorial, and abstract. Each level is designed to prepare students for the next. The use of representations, such as tape diagrams, is aligned with the Common Core State Standards.

Zearn Math CS PD

In the Zearn Math CS PD, teachers explore each unit, or mission, through word problems, fluencies, and Small Group Lessons, as well as conduct collaborative analysis of student work and strategies for problem solving. CS PD is structured around each mission's big mathematical idea, visual representations to scaffold learning, and strategies to address unfinished learning from prior grades and preparation for future learning.

Exhibit 1. CS PD Components and Outcomes



Study Approach

To investigate whether and how CS PD led to teachers PCK growth, we took a case study approach, aiming to look at a handful of teachers' diverse experiences with CS PD and the Zearn Math curriculum in different schools and districts.

We identified eight teachers in third to fifth grade, at four different schools in three districts, across three different states. We collected several types of data for each of these case-study teachers: think-aloud interviews with teachers to learn about their level of PCK before and after they had participated in CS PD for a fractions "mission" and implemented that mission in the classroom; observations during CS PD sessions and of one lesson on fractions; interviews about teachers' instruction and experiences with CS PD; interviews with administrators at teachers' schools; and a brief background survey for all the grade 3–5 teachers in case study teachers' schools.

In our analysis, we first characterized teachers' PCK level and growth. We then performed a cross-case analysis of PCK, teachers' CS PD experiences, their observed instructional practices, and what teachers reported about their instruction and experiences with Zearn Math, including CS PD. Finally, we developed detailed case study profiles and generated hypothetical explanations to make sense of each teacher's PCK level and growth and the overall picture of their backgrounds and instructional contexts.

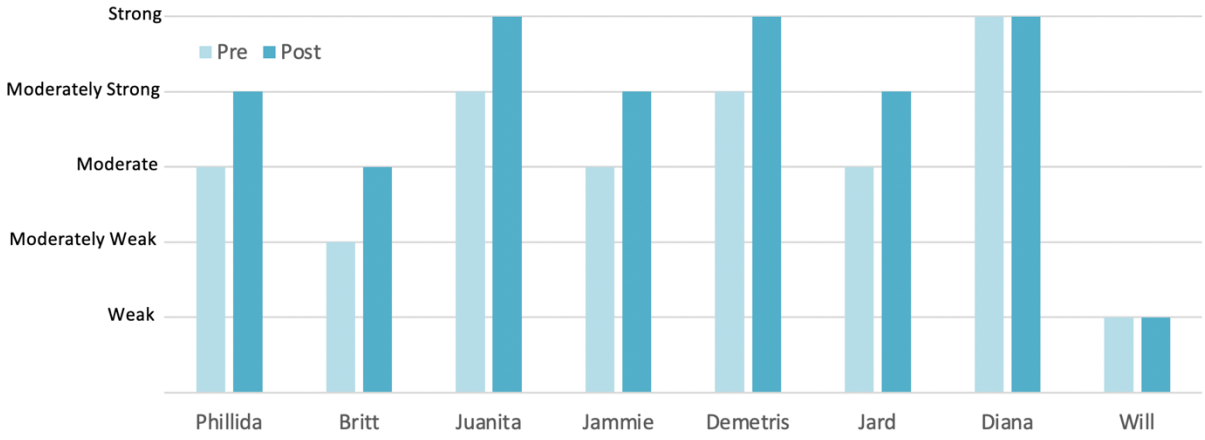
Research Questions

1. What PCK did teachers learn over the course of a year of teaching Zearn Math and where do they say they learned it?
2. What is the relationship between PCK and teachers' implementation of Zearn Math?
3. How can Zearn improve its CS PD?

Teachers Gained Pedagogical Content Knowledge (PCK)

We found that 75% of teachers in the study gained one level in PCK, as gauged by think-aloud cognitive interviews conducted before and after teachers engaged in CS PD for fractions for their grade level.

Exhibit 2. Teacher PCK Growth from Pre to Post Interviews



- Teachers' levels of PCK ranged from weak to strong with the mode for the end score at moderately strong. These levels were specific to knowledge needed to teach Zearn Math according to the Zearn Math logic model.
- The two teachers who did not grow in PCK, Diana and Will, were at levels strong and weak, respectively, in the initial interviews. Thus, Diana had ceilinged out at strong and Will seemed to have gained little from any of the Zearn resources, and his teaching was focused on accurately carrying out procedures only.

> *"I like that Zearn provides several strategies to get to the answer…you see the problems; you see what you need to hit on and stress the first time around."*
>
> *- 3rd Grade Teacher*

> *"[CS PD] has pushed me in some areas where, normally, I'd say, 'You have the standard algorithm down. You're good.' It pushes them and pushes me to show it to them in a different way…I definitely think it showed me some different strategies for students to use."*
>
> *- 4th Grade Teacher*

CS PD Materials and Implementation

The written CS PD materials included features that addressed almost all key areas intended to support teacher PCK learning, with one caveat: there was less support for teachers to respond to students' in-the-moment problem solving. This is, however, a challenge for any written instructional materials. As implemented, CS PD varied with facilitation and teacher engagement.

- The presentation of the development of a big idea was a strength of CS PD and several teachers said it was helpful
- Teachers were prompted to engage in authentic problem solving, but only half of the teachers we observed engaged as intended
- The enacted CS PD was job-embedded in several ways and almost all teachers said it connected to their practice
- Almost all the teachers collaborated actively and said CS PD supported their collaboration
- The CS PD materials provided some strategies to address student thinking and prompts for teachers' authentic problem-solving, but they needed to be engaged with and built upon by facilitators and teachers, which did not always happen
- In addition to CS PD, teachers reported learning from a variety of Zearn Math resources, indicating that the curriculum materials and their implementation are important sources of learning

> *"We tell kids to show their work, but Zearn is very specific about how to show their work and being able to make their thinking visible. I think that's something that I've learned is really valuable for me, to be able to see how they're solving it. I'd say probably that and just requiring them to explain more, and agree with each other, and have some discourse when they're actually solving the problems I think has also changed."*
>
> *- 4th Grade Teacher*

Zearn Math in the Classroom

Teachers' implementation of the Zearn Math curriculum varied in fidelity to the logic model's features. Teachers enacted many of the envisioned practices, while largely maintaining the traditional teacher role of controlling the classroom.

- All classrooms had a positive social atmosphere and good relationships among teachers and students
- In whole class discussions, teachers seemed to have a narrow view of agency. With some exceptions, discussions were mostly teacher-driven with less space for student thinking than envisioned
- Most teachers struck a good balance between the digital and face-to-face components of the curriculum
- Most teachers tried to make connections with student prior knowledge
- Half the teachers used remediation strategies in addition to the digital remediation supports
- In small group work and as students worked on the digital component, the teachers showed evidence of giving precise, timely, and safe feedback

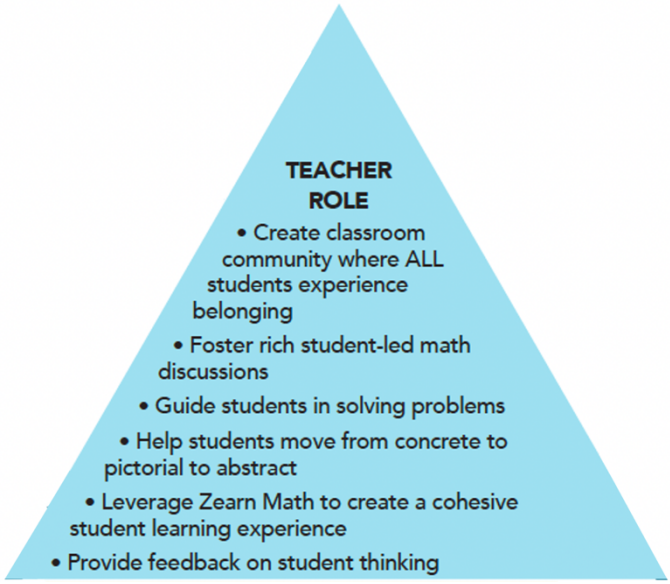


Exhibit 3. Zearn Math Classroom in the Logic Model

> *"I want them to just stay focused…I just want to hear everybody's voice, and I think everybody voice matters. I try to keep a calm setting where everybody can focus and get what they need to be done without being distracted."*
>
> *- 4th Grade Teacher*

> *"Classroom management is very important too, because no matter how great you are at teaching a concept, or a skill, you still have to have classroom management and keep…the students engaged."*
>
> *- 3rd Grade Teacher*

Teachers' Background, Instruction, Engagement, and Growth

The in-depth individual case study analysis yielded a range of different patterns with regard to teacher experience, instructional approaches, attitudes toward and engagement with the Zearn Math curriculum and CS PD, and PCK growth. Across this variation, we identified some key themes.

- Zearn Math (including curriculum, materials, and CS PD) appeared to help most teachers to some degree with regard to understanding student thinking, being attentive to students' roles, and adopting new or different instructional strategies
- There seemed to be a tension between centering student thinking and traditional instructional approaches; teachers tended to do most of the cognitive work and in many cases used behaviorist approaches to manage student behavior and attention
- Teacher background, training, and experience appeared to shape teachers' CS PD engagement, implementation of the Zearn Math curriculum, and classroom management in ways that suggest differentiation in support and PD is important
- Teachers' dispositions of interest and dedication to students' thinking and background may have played important roles in how they facilitated student engagement

> *It seems Jammie benefitted both from the curriculum and the CS PD. She was a very experienced teacher, who said she valued conceptual learning, and who seemed to embrace the Zearn Math curriculum and ideas. She was familiar with the big ideas and worked actively with them in planning and implementation. She said Zearn Math had helped her learn new and different strategies to respond to different types of learners. She said Zearn Math made student thinking visible, which helped her understand it better. While Jammie's PCK was moderate-strong, her classroom seemed to be characterized by traditional classroom management approaches with a fair bit of teacher control mixed with some student choice. Her own characterization of her instruction was more student-focused than what we observed in her classroom. It's possible that her PCK was limited by a more traditional, teacher-centered paradigm in the classroom.*
>
> *- Excerpt from 4th Grade Teacher Case Profile*

Recommendations

Our case study findings, while limited in scope, point to themes in professional development and instructional change that prior research and educational reform efforts have also grappled with. In our recommendations to Zearn for how to improve Zearn Math CS PD, we acknowledge the solid foundation CS PD and the Zean Math Curriculum have in place for supporting teachers' PCK learning and focus on three areas of improvement that are also relevant to others in the field.

- Focus PD more on how to support a variety of students' thinking, including displays of "preconceptions" to help teachers when they are "thinking on their feet"
- Differentiate PD to meet the needs of a range of teachers with different backgrounds, dispositions, and knowledge
- Increase the focus of PCK-oriented PD, such as CS PD or other curriculum-specific PD, on supporting student agency and teacher caring

Introduction

This report describes the Zearn Math Curriculum Study Professional Development (CS PD) study conducted by SRI Education and TERC. The study was funded with two grants, from the Carnegie Corporation of New York and the Charles and Lynn Schusterman Family Foundation, and began in August 2019. The focus of the study was to determine to what degree CS PD leads to teacher growth in pedagogical content knowledge (PCK), which is the foundation for high-quality implementation of the Zearn Math Curriculum. To investigate this question, we conducted multiple-case study research of eight teachers in four different schools located in three districts in different parts of the United States. The study included PCK-assessment interviews with teachers, observations of CS PD sessions and classroom lessons, as well as interviews with teachers about their experiences with CS PD and implementation of the Zearn Math Curriculum. We also conducted a brief teacher background survey of all the teachers in the case study teachers' schools. This report presents the study design, methods, analysis, findings, and a discussion about the implications and recommendations for CS PD.

Study Goals and Research Questions

Elementary school teachers often struggle when it comes to teaching mathematics, particularly mathematics as addressed in the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) or current state standards, which focus on specific mathematics practices and emphasize conceptual understanding (e.g. Swars & Chestnutt, 2016). Sometimes, this struggle is related to teachers' history in learning fractions; other times it relates to the small set of pedagogical strategies they have to address this topic. Yet another issue may be the lack of knowledge about students' thinking about fractions. Simply adopting a new curriculum rarely provides enough support for teachers to acquire a deep understanding in any of these three areas (Kennedy, 2005), nor is scripted teaching an answer when students are presenting their own, perhaps unique, reasoning about content. Teachers need high-quality professional development that is linked to the use of high-quality instructional materials (Ball & Cohen, 1996). Zearn developed CS PD to meet this need by explicitly addressing teachers' pedagogical content knowledge, a concept with decades of research behind it and from which this study draws. This study examines to what degree teachers develop their pedagogical content knowledge for teaching elementary school mathematics, specifically for teaching fractions, which is known to be a challenging topic in elementary school and one linked to students' later success in mathematics. We also explore possible explanations for patterns in teachers' PCK growth.

We designed the study to investigate how CS PD supports teacher PCK growth, teaching practice, the observed relationship between them, and what aspects of CS PD could be improved to best support growth in PCK and classroom practice aligned with the Zearn Math model. Our approach was to first develop a research-informed professional-learning logic model

describing the features of CS PD and how they hypothetically lead to the pedagogical content knowledge necessary for realizing the Zearn Math instructional model and characteristics of the ideal classroom engaged in Zearn Math (see Appendix B). We then designed a case-study research approach to explore teachers' experiences with CS PD and their implementation of the Zearn Math curriculum. Our goal was to develop the following products:

- Logic model graphic and explanatory document
- Report on eight case study teachers, their pedagogical content knowledge, experiences in CS PD, their implementation and reflections on the Zearn Math curriculum, and possible explanations for patterns in their PCK levels and growth
- Recommendations to improve CS PD materials

By the 1980s, research on teacher learning had not uncovered any relationship between teachers' math knowledge and successful outcomes for their students. Nor had studies of teachers' general pedagogical knowledge found any effect. In 1986, Shulman (1986) wrote about a form of teacher's knowledge that had not been considered which he termed "pedagogical content knowledge (PCK)." He explained that there is subject-matter-specific knowledge and subject-matter-specific pedagogy, and that these two are intertwined, resulting in PCK. This work was built upon by a generation of his former graduate students (e.g., Cohen & Grossman, 2016). PCK is currently thought of as a bundle of several kinds of knowledge: of relevant mathematics content related to teaching, of student thinking around this content, of pedagogical strategies for use with students learning this content, and of the curriculum being used to teach this content (Hill & Ball, 2009).

The content we consider in this report is fractions. A well-known practitioner-oriented book characterized fractions as "hard to learn, hard to teach" (Barnett-Clarke et al., 2003) and many teachers still think of fractions this way. However, research over the past two decades has provided a range of methods for teaching fractions that provide better leverage for successful learning (Empson & Levi, 2011; Steffe, 2004).

The Zearn Math curriculum approaches fractions by leading students through a series of representations of fractions: concrete, pictorial, and abstract. Each level is thought to prepare students for the next. This approach is consistent with Bruner's theory of instruction (1966) and has been popularized as the approach taken in *Singapore Math* (Leong et al., 2015). The use of representations such as tape diagrams is also aligned with the Common Core State Standards. This sequence plays out in practice, as reported by teachers in these PCK interviews, in the following ways:

- At the concrete level, teachers report acting out situations, showing photographs of real-world objects with fractional parts indicated, or students using fraction manipulatives

- At the pictorial level, teachers report that students draw diagrams, use tape diagrams, or a number line. In some cases, students are instructed to use specific representations; and in some cases, they are asked to choose a representation to solve a problem
- At the abstract level, students write number sentences or use "number bonds" to solve problems. Number bonds, perhaps, bridge the pictorial and abstract and provide representations to enable students with multiple means to use and interpret number sentences

The study was designed to address three overarching research questions[1]:

1. What PCK did teachers learn over the course of a year of teaching Zearn Math and where do they say they learned it?[2]
2. What is the relationship between PCK and teachers' implementation of Zearn Math?[3]
3. How can Zearn improve its CS PD?

To address Research Question 1, we conducted think-aloud interviews (Charters, 2003) with teachers to elicit their PCK knowledge, on two occasions: before they participated in the fractions-focused CS PD and again toward the end or after they had completed the fractions mission (a curricular unit on a particular topic). To address Research Questions 2 and 3, we used a multiple case study approach (Stake, 2006; Yin, 2017), in order to study the relationships between teachers' CS PD experiences, implementation of the Zearn Math curriculum, and their PCK growth. The data for the case studies included the PCK interviews, observations of teachers' participation in CS PD, observations of one of their lessons, interviews with teachers about their experiences with CS PD and the Zearn Math curriculum, and the teacher background survey.

[1] These research questions were modified from the original ones proposed. First, we learned during the course of the study that teachers struggled with attributing what they were learning to CS PD specifically, so we broadened the question to include the Zearn Math curriculum and teacher's experiences implementing Zearn Math. Second, we removed the question about teachers gaining mathematical knowledge, as we were not able to gather sufficient responses to the LMK teacher survey.

[2] This question departs from the original question, which focused solely on CS PD as we saw that teachers reported many different aspects of Zearn Math as sources of their learning, including Zearn Math student materials, online learning tools, teacher manuals, PD, and also their own teaching. At times, teachers referred to "Zearn," undifferentiated by particular resources, but as a bundle contributing to their learning. We believe that this broader question will result in useful findings for the Zearn Math team, about the entirety of teachers' sources of learning, and, of course, what they learned.

[3] This question was simplified because of the difficulty in parsing out what learning was attributable to what source, yet patterns of connections between PCK and classroom implications were detected as "hypothetical explanations."

Research Staff

Four researchers from SRI and TERC collected data and conducted the analyses. Jennifer Knudsen is a senior mathematics education researcher at TERC. She has directed a series of research projects on teacher PD, funded by the National Science Foundation. With 30 years of experience in curriculum design and teacher PD, she has focused on helping teachers develop pedagogical content knowledge for high-level disciplinary practices such as mathematical argumentation in topics from fractions to transformational geometry. She is lead author of the book, *Mathematical Argumentation in Middle School: The What, Why, and How* (*Knudsen et al., 2017*). Patrik Lundh is a senior education researcher at SRI Education where he leads and conducts qualitative research in science and mathematics education. With a background in cultural anthropology, he has expertise in ethnographic and case study methods and is particularly focused on sociocultural practices in schools and classrooms. Mindy Hsiao is a research associate at SRI Education. She is a contributing author to evaluation reports on math professional learning communities, coaching, and computational thinking curriculum. She holds a Master's in education policy from the University of Washington, a Master of Arts in teaching from National Louis University, and a BA in psychology and a BA in creative writing from Northwestern University. Prior to SRI, she taught sixth grade in Chicago Public Schools. Daniela Saucedo has experience with qualitative and quantitative research methods as a research associate with SRI Education. She has contributed to numerous reports on topics ranging from broadening participation in STEM to improving outcomes for students who are off track to graduate from high school; clients include the Department of Education, the National Science Foundation, and the American Museum of Natural History. She earned her BS in economics from Duke University and has a passion for addressing systemic education inequality.

1. Methods

In this section, we describe our approaches to selecting schools and teachers, our research design and research instruments, and our approach to data collection.

1.1 Sampling Approach

To select teachers, we worked with the Zearn organization to identify three schools in three different school districts. Our goal was to study schools that varied in terms of grade level (across Grades 3–5), prior content knowledge, teaching experience, and professional learning community groups as well as the diversity of students they teach. We identified four schools in three districts in three different states and a total of eight case study teachers. We used a maximum variation sampling approach (Flyvbjerg, 2006) to ensure the cases would offer a range of experiences and scenarios which makes for more opportunities for rich explanations. Our sample looks as follows (we use pseudonyms for teachers, schools, and districts; also, the information in the table is drawn from both survey and interview data):

Exhibit 4. Case Study Teacher Sample

	Phillida	Britt	Juanita	Jammie	Demetris	Jard	Diana	Will
Grade	3	3	4	4	4	4	5	5
School	San Lorenzo	Fullerton	Fullerton	Chabot	Laurel	San Lorenzo	San Lorenzo	Fullerton
District	A	B	B	C	C	A	A	B
Years Teaching	2	1	4	12	3	2	14	2
Years Teaching Math	2	1	4	12	3	2	1	2
Elementary School Certified	Yes	No	Yes	Yes	Yes	Yes	Yes	n/a
Math Certified	No	No	Yes	No	No	No	No	n/a

1.2 Constructs Table

To translate our research questions into a research strategy, we created a constructs table (see Appendix I) in which we first broke down each research question into constructs to be investigated. We then aligned each construct with a data source[4]. We used the constructs table to guide the development of each instrument to ensure that the interview and observation protocols were targeted to elicit the right data to answer the research questions. It also helped to ensure that each research question was adequately covered.

1.3 Case Study Design

Our case studies were designed to collect multiple data about each teacher to allow us to describe and explore the relationships between their experiences in CS PD, their PCK growth, and their experiences implementing the Zearn Math curriculum. The goal was to make sure that data collected about each teacher was consistent across teachers to allow us to look for themes and variation across the cases. Another goal was to use multiple data sources to inform rich case profiles of each teacher so that we could explore possible explanations for the relationships between PCK growth and teacher backgrounds, experiences, and practices.

1.4 PCK Interviews

To learn about case study teachers PCK level and growth, we interviewed them twice; in December 2019 and again in the late spring of 2020. The interviews were about 30 minutes

[4] Our original design included the Mathematical Knowledge for Teaching assessment. However, we only received responses from six teachers, so we decided to discontinue this data collection.

long and began with a problem that students would learn to solve at each teacher's grade level, along with an incorrect student answer portraying a commonly occurring misunderstanding. See Appendix C for the interview protocol.

The first interview was done as a "think-aloud" (Charters, 2003) with symbolic math expressions displayed on a shared screen. The prompts in the protocol, used when teachers did not cover the idea in their thinking aloud, served to capture the different aspects of PCK—content, students, pedagogy—in addition to where they thought they learned each explanation they gave. This last prompt was prefaced by saying that we often don't know exactly where we learn something, but they should just give their best conjecture about the source.

The second interview followed this format but also included a video prompt of a student solving a word problem using paper and pencil diagrams as well as symbolic math expressions.

1.5 CS PD Observations

For the observations of the CS PD sessions, we developed an observation protocol which included two components. One was for the observer to take time-stamped, running notes, documenting what facilitators and PD participants said and did as well as other, relevant contextual factors. The other component of the protocol was a structured observation debrief in which the observer wrote up summaries, based on the running notes, that addressed several categories aligned with constructs from the constructs table (see Appendix E). We observed one CS PD session for each teacher, except one (the reason for that was we only had two observers on site at a school with three case study teachers in three different grades whose CS PD sessions took place simultaneously in three separate rooms).

1.6 Classroom Lesson Observations

The observation protocol for the lesson observation was similar to the CS PD observation protocol, only it focused on different constructs. We observed one lesson for each teacher, except one, who fell ill at her scheduled observation, which occurred just before all the schools shut down due to COVID-19. See Appendix F for the protocol.

1.7 Teacher Instructional Interviews

We developed semi-structured interview protocols with questions that were aligned with the constructs in the constructs table. Semi-structured means that the protocol was organized around specific questions to give the interview a consistent structure and ensure that key areas were covered, while at the same time allowing for flexibility for interviewees to elaborate and for the interviewer to ask follow-up questions. The interviews focused on teachers' experiences with the CS PD, implementation of the fractions mission, their beliefs and ideas about instruction and the Zearn Math curriculum, and how they evaluated their own PCK and general capacity and confidence in teaching math. Some interviews were conducted in person and some on the phone or via Zoom. All interviews were audio-recorded and transcribed. See Appendix D for the protocol.

1.8 School Administrator Interviews

The purpose of interviewing the administrator at each of the three schools was to learn about the school context for using the Zearn Math curriculum as well as the CS PD. We developed a brief, semi-structured protocol (see Appendix G), focused on the use of Zearn Math in the school and the context of the CS PD. The interviews were audio-recorded and transcribed.

We also tried to contact three school administrators during the summer to ask about their handling of the COVID-19 crisis and their plans for the fall. We were only able to talk to one of the administrators. One had retired and another was not able to participate. We did not use an interview protocol for this discussion. We simply asked the administrator to share their experiences about the impact of COVID-19 on the school and the school's plans related to COVID-19.

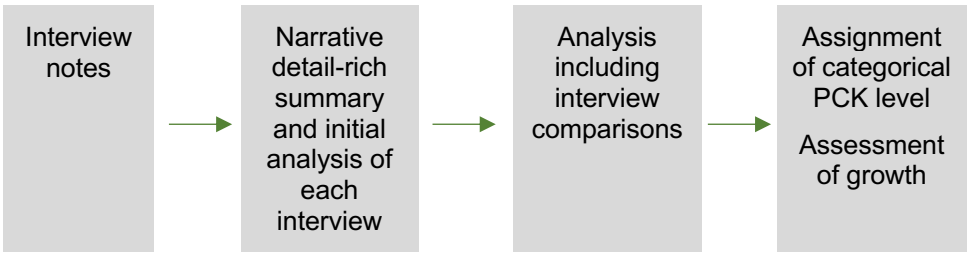
2. Analysis

In this section, we describe our approaches to processing and analyzing the different kinds of data, including the PCK interviews, CS PD and lesson observations, teacher interviews, and teacher background survey.

2.1 PCK Interviews

Data was captured through detailed notes taken concurrently with the interview. The notes for each interview were then composed into a narrative, preserving detail of teachers' reported thinking (Mishler, 1986). These narratives were given structure by writing an analytic memo on each one, retaining details from the narrative as examples as often as possible. Then a comparative analysis was made between the first and second interview, attending to features of the project's logic model and also to the aspects of PCK we address in the introduction. While more compact than the initial narrative, these were also written to preserve details of teachers' thinking as well as the interviewer/researcher's reflections on what was said. Each teacher's PCK was evaluated qualitatively, relying on the literature (e.g., Empson & Junk, 2004) on what an elementary teacher needs to know about content, instruction, and students' thinking for Grades 3–5 fractions; the context of Zearn Math's instructional and curricular approach; and the researcher's judgment as an experienced designer of professional development aimed at increasing PCK. They were then assigned a categorical rating from "weak" to "strong" within the context of teaching Zearn Math. In other words, the ratings are not comparable with what would constitute ratings of teachers using a different curricular and pedagogical approach. This is a somewhat unusual stipulation but provides a better picture of PCK relevant to Zearn Math than would a generic rating. PCK growth from the first interview to the second was determined by further analysis of the interview comparisons (see Exhibit 2).

Exhibit 5. Diagram of the Flow of Multiple Analyses of Interviews



2.2 Observations of Curriculum Study PD and Classroom Lessons

The observation process was the same for both the CS PD sessions and the classroom lessons. The observer took time-stamped running notes during the observation. After completing the observation, the observer reviewed the notes and wrote up summaries in a structured debrief form in categories aligned with relevant constructs in the constructs table.

2.3 Teacher Instructional Interviews

The teacher interviews were audio-recorded and transcribed. Analysts then read through the transcript and wrote up descriptions, summaries, or claims in a table, under categories aligned with constructs in the constructs table.

2.4 Teacher Background Survey

We received a total of 27 teacher background surveys from Grade 3–5 teachers at the four schools. Our first cleaning step was to remove any records with no consent or duplicate data. For the duplicate entries, we choose to keep the record with the earlier date. We also cleaned the school names to be consistent across the survey. Finally, we generated tables of all the data for the survey questions, and also created teacher profiles for the case study teachers for each school so that we could compare case study teachers to the overall sample as well as the sample at their school. The final survey sample was 23 teachers in Grades 3-5 from all four schools.

2.5 School Administrator Interviews

We reviewed the interview transcripts and wrote up summaries for each school to include as context for analyzing the case study data.

2.6 Case Study Analysis

To analyze the teacher case study data, we first created a case analysis table containing the summaries and descriptions from all the data organized so that the analyst could compare categories across all teachers as well as see all the data for each teacher. The analysis proceeded in three steps: (1) creation of teacher case profiles; (2) cross-case analysis; and (3) within-case analysis and the generation of hypothetical explanations.

2.6.1 Creation of Teacher Case Profiles

Teacher case profiles were written with low-inference preliminary analyses for the following categories:

- Background and Instruction: Description of the teacher's background and approach to instruction
- Zearn Math Impact on Instructional Practices: Teacher's report on how Zearn Math has changed their instructional practices
- Teacher CS PD Engagement: Teacher's engagement with CS PD, based on the observations, including what appeared to work well and what appeared to be challenging
- Implementation of Zearn Math Lesson: Description of what worked well, what was challenging, and ways in which the lesson aligned with teacher role and student experience as outlined in the Logic Model, based on observation and interview

2.6.2 Cross-Case Analysis

The purpose of the cross-case analysis was to identify potential themes and to describe the variation across the cases with regard to the key research constructs we investigated. Using the case analysis table, we went through one construct at a time (e.g., teachers' interview responses to how they approach whole class discussion), reviewing the data for each teacher and writing summaries addressing the following: themes (ideas identified in more than half the cases); the variation or range of ideas within the theme, in addition to the theme or in lieu of an identified theme; notable exceptions, outliers, or examples. We then described these interpretations in relation to the key ideas articulated in the Zearn Math Logic Model.

2.6.3 Within-Case Analysis: Generation of Hypothetical Explanations

The purpose of case study research is not to generate findings that are generalizable to a broader sample, which is not possible with a small number of cases. The strength of the case study approach is rather in understanding the relationships, dynamics, and contexts at play in a particular phenomenon or problem. Case studies allow for the close examination of a phenomenon and produce findings that can generalize to theory (Eisenhardt & Graebner, 2007) and to build explanations for a phenomenon by generating hypothetical explanations (Yin, 2017). In this study, we focused on building hypothetical explanations related to the research question: What is the relationship between PCK and teachers' implementation of Zearn Math? These explanations are characterized as hypothetical, since they represent plausible explanations about patterns in the data rather than absolute truth statements about causal relationships.

For this analytical process, our team of four researchers was assigned in pairs to each case study teacher so that each case was initially reviewed by two researchers in different combinations. Each researcher first worked independently to carefully review all the data for the

teacher case; including CS PD observation, lesson observation, teacher interview, and teacher PCK interview findings. Based on these data, we generated one or more explanations that responded to the research question. Across all eight cases, we generated a total of 26 explanations. We then met to discuss and reconcile any contradictions or disagreements about their interpretations as well as to compare explanations across cases and clarify differences and similarities between them. We also documented prior research that supported or contradicted explanations as an additional check on their strength.

Finally, we reviewed all the cases and explanations to identify themes and variations among them. We looked across all the explanations generated to look for common themes, variations, and ranges within the themes, variations and ranges among the explanations, and if there were no common themes and any notable exceptions or outliers.

3. Findings

In this section, we describe several aspects of our findings.

- A summary of the school contexts for the three schools from which we selected the case study teachers
- A brief discussion about the impact of COVID-19
- The overall findings from the teacher background survey
- Case study findings, in two main parts:
    - An overall summary of the PCK findings, the cross-case findings, and the hypothetical explanations we generated
    - Detailed descriptions of the cross-case findings and hypothetical explanations for each case study teacher's PCK level and growth. Additional details about the teacher cases are provided in Appendix A.

3.1 School Contexts

3.1.1 Fullerton

Fullerton is one of eight schools in a mid-size U.S city that is part of a private, Christian school network where students attend using public vouchers. The school serves PK–8 grades and has approximately 500 students; 96% are African American. There are two grade 3, two grade 4, and one grade 5 teacher.

According to the administrator, the school and the network had used Eureka Math for some time. She said one of the teachers came across Zearn Math and asked for permission to use it, which led to Zearn Math becoming used across the school network in K–5. Eureka and Zearn Math were the two curricula the school uses. They had been using Zearn Math for the past 2 years.

Administrators managed the CS PD scheduling. The administrator said she tracked the teachers' progress in their grade books to ensure they were staying within the scope and sequence for the year. When teachers were close to finishing a Zearn Math mission, she would schedule time for teachers to do the next CS PD session. This usually took place during their check-in time, given teachers' large workload. She said teachers reported the CS PD had been going well:

> From the feedback from my teachers, it's helped them. In previous years, we hadn't done the Zearn PDs; and by doing it this year, they said that they felt more prepared. They better understand what's expected of them in each mission, and that way, they can teach it even better for their kids.

Teachers attended some kind of math PD every week and collaborated by grade level, except for the fifth grade, which had only one teacher. The collaboration had been important and beneficial, according to the administrator.

The school's goal was to have their students do at least four Zearn Math lessons per week. She said that students varied widely in their math abilities, some lagging years behind grade level, which posed challenges for teachers, and that Zearn Math's differentiation capacities had been very helpful in this regard.

3.1.2 San Lorenzo

San Lorenzo is a public PK–5 elementary school in a small city, serving approximately 500 students who are 54% African American, 23% Hispanic, and 18% White. Sixty-six percent of the students received free and reduced lunch.

The curriculum coach said that the school was flagged for state intervention, which means the school frequently received visits from the state to observe, and the school was closely monitored. The school had two Grade 3 teachers, one Grade 4 teacher, and one Grade 5 teacher.

According to the curriculum coach, the school had been using Zearn Math for the past 3 years. They also used the Eureka curriculum. She said Zearn Math was good about providing an overview of the program at the beginning of the year. The school administration used the mission assessments to check progress, in particular making sure that students and teachers were using the correct vocabulary. They also used Edulastic Interactive Formative Assessment which is computer-based. In the RTI (Response to Intervention) classes, teachers also used other resources such as Khan Academy.

The curriculum coach facilitated the CS PD sessions. She met with teachers in their grade level team PLCs and reviewed the mission and the assessments, worked through the problems, and sometimes administered the assessments to the teachers so that they experienced what students would experience. She said some teachers were not very engaged.

> Well, you know, people like to do things their own way. When somebody comes in and says, “This is the way we’re going do it,” they’re not as receptive as you would want them to be. I mean, sometimes it gets a little tough but like I said, we’ve been doing this for 3 years. They know this is the way we’re doing it. It goes fine. They sit there and—now taking the test. I’ve had some people just no, they’re not going to do it.

She also said that teachers who had implemented Zearn Math in prior years found the CS PD repetitive. Once a month, teachers also participated in vertical team meetings where all math and English language arts teachers met to discuss alignment.

When the Zearn Math curriculum was introduced to the school, the school originally did not have computers for each student, so they had to wait on full implementation. But now they do. Some students struggled with basic computer skills. The curriculum coach said the Zearn Math implementation in the 2019–20 school year was “excellent” because the teachers were more familiar with the curriculum, and it was now used schoolwide for all students. She said the school had benefited from using Zearn Math:

> I believe our test scores have gone up in math in the three through five [grades]. We ’re not going to have any results from this year. But as far as with you giving us the foundational skills, that when a student is experiencing trouble and they are going back and addressing those skills, I think the scores have come up and their understanding is a lot better and so is their problem-solving. They know different steps to problem solve. It’s not just one step or one strategy to solve a problem. They have multiple strategies.

Many students in the school, particularly in lower grades, struggled with basic math skills, according to the curriculum coach:

> That’s another thing too. In the first grade and second, they don’t know how to—well subtraction’s a difficult thing in second grade. First grade is just the addition. Now I believe that Zearn starts off with mission one—it’s a little bit more difficult for first graders because I think they have a lot of word problems. Some of our first graders cannot read until later on. Anyway, with the multiplication facts in third grade, they worry about that and the division up into fifth grade. Main thing with second grade is, I forget what you call it. I call it regrouping or you’re borrowing a number or something like that. They have a lot of difficulty with that in subtraction.

3.1.3 Laurel and Chabot

Laurel and Chabot are two elementary schools in a mid-sized U.S. city. They are managed through a national nonprofit organization that partners with districts to turn around low-performing schools. The organization is also partnered with an independent, accredited graduate school of education with a teacher residency program and a principal fellowship program.

Laurel Elementary is a PK–6 public charter school serving 440 students who are 97% Hispanic and 3% Black. Ninety-nine percent of the students receive free and reduced lunch and 27% are English language learners.

Chabot Elementary is PK–7 public charter school serving 637 students who are 98% Hispanic and 2% White, Black, Asian, Native American, or two or more races. Ninety-seven percent of the students receive free and reduced lunch and 36% are English language learners.

According to the administrator, who was the managing director of instruction for math at the two schools, Zearn Math was the primary curriculum both schools used. The curriculum had been used at Laurel for 2 years and Chabot for 3; however, this was the first year both schools strived to implement it with fidelity. The administrator's role was to align Zearn Math to the state standards, develop frameworks and structures for how Zearn Math would be implemented in the classrooms, coach math teachers, and analyze the student data for areas of improvement for teachers.

The administrator said she appreciated Zearn Math and that it had benefitted the teachers in a few ways. The scripted lessons helped them with their questioning strategies, especially the newer teachers, and the CS PD focus on content development and how ideas progress was informative for teachers. She also said she appreciated that the Zearn Math digital component has rich data for teachers to use and understand where students are struggling.

The administrator also noted a couple of challenges with Zearn Math. Teachers struggled with building routines for small group lessons, whole group lessons, and the digital portion. Further, a majority of their students were significantly behind grade level, which meant remediation was a main focus for the schools. Both of the schools had weekly data meetings where they analyzed student work and looked for gaps in understanding. During those meetings, they would plan how to reteach for the following Friday when they remediated students.

3.2 COVID-19

The special circumstances in which the case study teachers found themselves cannot be overlooked. Schools closed their buildings in late March or early April due to the novel coronavirus pandemic. Teachers began engaging with their students through distance learning, using digital resources such as those found in Zearn Math or supported by learning management systems such as Google Classroom. Some engaged through paper-based assignments. When teachers were interviewed in the spring, mostly in May, their unique experience of participating in CS PD was over or nearing its end. Many stated that they felt disconnected from their students and teaching. A couple of teachers said their answers in the second PCK interview may not have been as complete as they had been in the first interview, due to this disconnect. However, the evidence showed otherwise for most of the teachers.

One teacher's instructional interview took place after the school closures, and she was able to speak to us about her experience teaching during this time. She said that the pandemic made it

more difficult to introduce new material. She was appreciative of Zearn Math's digital component because it allowed her students who had reliable internet access to continue their math. For her, the Zearn Math instruction that she would have taught in person was difficult for her to incorporate virtually, because her students were not able to be on the same schedule, so many students would be at different points of the lesson.

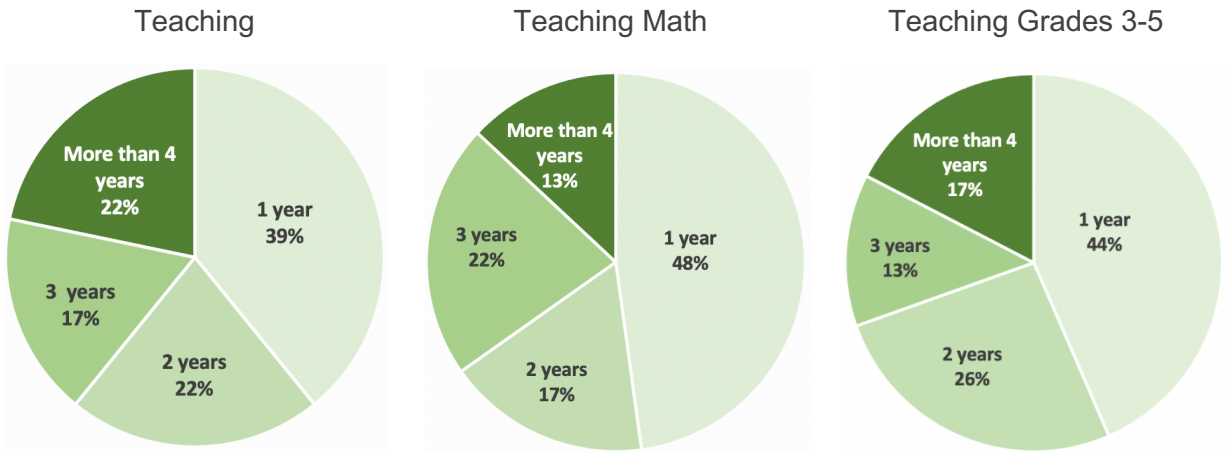
One administrator was able to speak about her school's plans for Zearn Math and the upcoming fall 2020 semester. Over the summer, some teachers worked on creating virtual content that would be available on Google Classroom for the school year so that teachers across the network would be on the same page. The administrator's school also became a 1:1 school and families were provided internet access through partnerships with local internet providers. Because of this, all students would be able to access Zearn Math's digital online component. The network planned on structuring their virtual math classes with at least 30 minutes per class where students would be working on Zearn Math online. This would allow teachers to check Zearn Math to see how many and how often students log on and where they are struggling. The school planned on continuing the CS PD sessions with a session in the summer using Zearn Math's CS PD videos. The administrator seemed hopeful about the digital aspect of Zearn Math because the students all had devices that they could access the content on from home.

3.3 Teacher Background Survey

Overall, 23 teachers responded to the survey. Most of the teachers were relatively new to teaching experience (n = 14), i.e., had taught for 1–2 years (see Exhibit 3). More than half of the teachers (65%) reported having taught math for 1–2 years and around 83% of the teachers taught grades 3–5 less than 4 years.

Most of the teachers (N = 16) had an elementary education teaching certificate. Many teachers (N = 18) reported participating in professional development related to math during the past 5 years.

Exhibit 6. Survey Findings on Teachers' Years of Experience



3.4 Case Study Findings Summaries

This section includes summaries of the PCK interview analysis, the cross-case findings, and the within-case findings, including the hypothetical explanations that we generated for each case. We provide more detailed findings in section 3.5.

3.4.1 Summary of PCK Findings

The table below summarizes, at a high level, the results of the analysis of two interviews designed to elicit teachers' PCK with regard to fractions.

Exhibit 7. High-Level PCK findings by Teachers (by Name) and Schools

	Phillida	Britt	Juanita	Jammie	Demetris	Jard	Diana	Will
Grade	3	3	4	4	4	4	5	5
School	San Lorenzo	Fullerton	Fullerton	Chabot	Laurel	San Lorenzo	San Lorenzo	Fullerton
PCK end-state level	Moderately Strong	Moderate	Strong	Moderately Strong	Strong	Moderately Strong	Strong	Weak
Growth	Yes	Yes	Yes	Yes	Yes	Yes	No	No
Teaching strategies based on multiple representations	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Teaching strategies in response to student thinking	Yes	No	Yes	Yes	Yes	Yes	Yes	No
Cited Zearn as source of learning	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Cited CS PD in PCK interview	No	No	Yes	Yes	No	Yes	No	No

The table shows that six out of eight teachers demonstrated growth in PCK, and all but one cited Zearn Math as a source of their learning. Three of those five teachers cited CS PD as a source of learning. Teachers' overall level of PCK in relation to Zearn Math (as described earlier) varied from weak to strong, with most teachers falling in the moderately strong category: this category was reflected in, for example, making one mathematical mistake or having only one teaching strategy to address the preconception given in the interview prompt. The table shows two important elements of PCK:

- *Regarding knowledge of content and curriculum*, all but one teacher displayed knowledge of multiple representations for fractions and teaching strategies that use them.
- *Regarding knowledge of student thinking*, all but one teacher was able to describe at least one additional teaching strategy in response to student thinking.

The number of strategies and their ability to describe them varied, with most teachers able to describe several strategies based on representations and fewer based on student thinking; however, the latter was an area of growth.

Details on each teacher's growth are found in section 3.5.3, Within-Case Study Findings

3.4.2 Summary of Cross-Case Findings

The cross-case findings describe the themes and variation we found in CS PD and lesson observations and teacher interviews *when looking across cases*.

Overall, we saw a picture of some contradictions, as is natural in comparing intentions in curricular materials, how people use those materials, and how they say they use the materials (Remillard et al., 2011). We found that the written materials fulfill the expectations set for them in the logic model, with one exception: taking the definition of responsive thinking implied by current research, the materials did not support teachers in thinking about how to respond to students' in-the-moment problem solving, which goes beyond prompting for different strategies students may use based on mathematical representations. This is a tall order for any set of written materials to fill. In practice, we observed that teachers occasionally did go to a deep level on students' thinking. In addition to CS PD, teachers reported learning from a variety of Zearn Math materials, an indication that the curriculum materials themselves are an important source of learning, as is teaching in and of itself.

Implementation, teachers' actual practice, varied in its fidelity to the logic model's features. Overall, there was evidence that teachers had created a positive classroom community. We saw that teachers' practices typically included, particularly in whole class discussions, some Initiate-Respond-Evaluate patterns, which can preclude opportunities for students to share their own problem-solving strategies or actively participate in classroom discussions (Sherin, 2002). On the other hand, teachers supported student thinking when students were working in small groups and individually.

This kind of contradiction between advocated methods and methods in practice is not uncommon and occurs for a variety of reasons: teacher's internal model of instruction may be derived from previous practice, or what they experienced as a learner, which holds a powerful influence over what they do as teachers using new methods and materials (Kennedy, 2005) Learning new teaching methods, or any new behavior, takes time.

3.4.3 Summary of Within-Case Findings

The within-case findings take the form of hypothetical explanations, which describe the most plausible explanations for patterns *within each case* with regard to the relationship between teachers' PCK, backgrounds, and implementation of the Zearn Math curriculum.

Unsurprisingly, the eight case study teachers varied considerably in experience, instructional approaches, attitudes toward and engagement with the Zearn Math curriculum and CS PD. As we tried to make sense of each teacher's PCK level and growth, we first and foremost put PCK in the context of what we saw and heard in the observations and interviews. Many factors contributed to teachers' PCK; the interconnections between school and classroom culture, teacher practices, teacher beliefs, and teacher learning are complex. Our task was not to pinpoint some particular factors that influenced PCK but rather to present a broad picture of each teacher and the relationships between these aspects of teachers' Zearn Math experiences. The hypothetical explanations for each of the eight teachers are described in detail below (section 3.5.3, Within-Case Study Findings).

Across most cases, Zearn Math (curriculum, materials, PD) appeared, to a limited extent, to help the teachers understand student thinking, be more attentive to students' roles, and have new or different instructional strategies. At the same time, we saw that there is a tension between the effort to center student thinking and traditional instructional approaches, wherein the teacher directs, reasons, and uses behaviorist approaches to manage student behavior and attention. In one case (see Jammie), the teacher's embrace of the Zearn Math curriculum and enthusiastic engagement in the CS PD seemed to be muted by the teacher's traditional instructional framework. But in many other cases, teachers combined some traditional practices and structures with overall attention on students and student thinking, which also seemed to go along with stronger PCK (see section 3.5.3, Juanita, Phillida, Demetris). At the same time, simply caring deeply about students didn't necessarily mean changes in practice (see section 3.5.3, Will).

Teacher background, training, and experience played a role in different ways. For example, Diana was a veteran teacher with strong PCK, but her instruction was in some ways traditional, with less student agency, and she appeared less engaged in the CS PD compared to other case-study teachers. She said she already drew from her own experience with her students rather than learning from Zearn Math, perhaps because of her many years of teaching. Will, a new teacher, was resistant to Zearn Math, but he mixed more traditional practices with ideas from Zearn Math and said he was still learning. In his case, the relative lack of experience might have meant that he relied on established practices, on one hand; and still, he had a long learning curve with regard to Zearn Math, on the other. In a more extreme example, Britt, who was a new teacher who also lacked formal training, might have had a particularly hard time being open to and adapting new practices which meant weak PCK and a reliance on traditional instruction.

Most importantly, we saw that a disposition of interest and dedication to students and their thinking and background seemed to play an important role, even for a new teacher and even in the face of traditional classroom constraints (see section 3.5.3, Demetris, Phillida), in particular after using Zearn Math for some time (see section 3.5.3, Juanita).

From the information teachers shared in the interviews, we learned that teachers had more in common by grade level than they did by school. Teachers of the same grade shared some characteristics which are taken as commonplace: Third-grade teachers expressed less confidence with mathematics. Fourth-grade teachers seemed most attuned to students' thinking than did teachers of the other two grades. Fifth-grade teachers showed more adherence to teaching algorithms only, though two of the teachers expressed that their teaching had changed in that regard over the year. In the Within-Case Findings below, we have noted for each teacher what we see as a growth area that CS PD could address. This may have implications for differentiation of the PD.

3.5 Detailed Case Study Findings

In this part, we describe in more detail our findings from the cross-case analysis and within-case analysis, beyond the above summaries. The Cross-Case Findings focuses on the CS PD (3.5.1) and teacher implementation of the Zearn Math curriculum (3.5.2). We describe the cross-case findings in relation to the key components in the Zearn Math Logic Model, in particular with regard to the CS PD components and outcomes and the expectations for what a Zearn Math classroom should look like.

The Within-Case Findings section (3.5.3) describes, for each of the eight case-study teachers, what we found about their PCK level and growth and our hypothetical explanations for the relationships between teachers' PCK, Zearn Math experiences, and instructional practices. For additional details about each case, see Appendix A.

3.5.1 Cross-Case Findings - Curriculum Study PD

Here we describe teachers' participation in and reported experience of CS PD based on PD observations and post-classroom observation interviews, respectively. We also include an analysis of the CS PD materials.

CS PD is designed to be used as job-embedded PD, focusing on the content teachers will be addressing next, and done with their grade-level colleagues. The PD is designed to take about 60 minutes and during that time, a facilitator, the teachers, and the videos have roles to play. CS PD is designed to be facilitated by a curriculum leader or teacher leader who has prepared by reading the facilitator and participant's guides and going through the videos in advance. The facilitator is to provide the teachers with guidance, stimulate discussion, and summarize the session, as advised in the facilitator guide. The videos call for pauses during which the facilitator instructs teachers to interact with their peers to solve problems and engage in other activities.

Five of seven observed sessions lasted between 50 and 60 minutes, during which teachers progressed through the CS videos, pausing in between to work in the participant guide (although this work was rushed during most sessions). The implementation of the facilitator and teacher-colleague roles varied. One teacher completed the session entirely on his own without a facilitator or any colleagues. One teacher had another colleague present, but neither of them took on the facilitator role. Three teachers (in three independent sessions) completed the session with one to two other colleagues and the school's curriculum coach; the curriculum coach helped set up the logistics of the session and had the facilitator's guide present but then instructed the observed teacher to do the actual facilitation. In their respective sessions, the teachers then walked their colleagues through the videos and prompted discussion using their participant guides, not the facilitator guide. In the other two of the seven teachers' observed PD sessions, both observed teachers were in the same PD along with one other teacher colleague and two residents; notably, the session was split across two 45-minute sessions. The teachers were from different schools whose math departments collaborate on curriculum; the two schools' sessions were combined over Zoom. The PD was led by the math department lead who followed the process described in the facilitator guide.

The participant guide is intended to play a key role in the PD. It contains portions of the teacher's guide to the curriculum and pages from the student curriculum. It is correlated to the video, but also contains planning prompts such as "skim through the small group lesson materials" and further prompts for what the teacher should identify and plan for using those materials. All observed teachers used the participants' guide during the CS PD sessions; when prompted in between videos, they would read through the guide or work through example problems in the guide. In most observations, work in the participants' guide was rushed because of time constraints—often teachers opted to spend more time discussing with each other than working independently in the guides.

According to the logic model, CS PD should increase teacher PCK through (a) leading teachers through an understanding of the progression of each big idea and its mathematical representations, (b) engaging teachers in authentic problem solving, (c) being content specific, (d) being job embedded, (e) leveraging peer teacher expertise and collective experience, (f) helping teachers understand student thinking and develop responsive teaching moves, and (g) supporting strong lesson planning. These elements are designed to help teachers deepen their math knowledge; deepen knowledge of how children solve problems, and gain conceptual understanding; have strategies to address students' misconceptions (e.g., using concrete materials, supportive visualization, alternative representations to help students); make connections between and across grade-levels; and apply what they have learned to plan effective lessons. Below we discuss how well these goals were met.

Goal: CS PD leads teachers through an understanding of the progression of each big idea and its mathematical representations

CS PD materials: The presentation of the progression of the development of a big idea was a strength of CS PD. The big idea and how the curriculum materials lead students to an understanding of it was part of the story of the video sequences. Additionally, the participant's guide contains a page-long description of the big idea and which representations students are to use as they develop an understanding of the big idea. Concrete-Pictorial-Abstract is the basis of the progression. How students would use each representation was the focus of some of the "solve the problem and think about..." prompts.

CS PD as observed: The big ideas were presented in the prerecorded videos. Each presented strategy for solving problems came with a clearly articulated rationale. Two teachers explicitly said they were eager to incorporate these ideas into their instruction. One teacher was especially well-versed in the terminology and rationale of Zearn Math and used it in the discussions with other PD participants. According to three of the four teachers who talked about the big ideas, the PD was helpful in that it showed different strategies for solving problems, from the visual to the conceptual.

Goal: CS PD engages teachers in authentic problem solving

CS PD materials: Experiences in authentic problem solving could vary, if we consider problem-solving as authentic when no immediate solution is apparent (Goos, 2004). There are several prompts for teachers to solve problems. Whether these were authentic problem solving would depend on a teacher's content knowledge going into the PD. Authentic problem solving would certainly be a way in which teachers develop their content knowledge for teaching, an aspect of PCK. The prompts for each problem often contained an additional prompt about how students might solve the problem, also leading to an authentic experience of a slightly different sort.

CS PD as observed: Teachers' engagement in problem solving varied. During observations, although most teachers read through the participant guide, only about half of the teachers actually worked through the problems at each pause in the video. Specifically, two teachers worked independently on problem-solving and then discussed their work with the group, while two other teachers worked collaboratively with their teacher colleagues as though they were students solving the problems. On the other hand, three of the teachers did not engage with the problem as if they were students: one teacher solved the problems independently in an adult role as if they were taking a quiz; one teacher would start to solve a problem, give up, and watch the solution in the video; and another appeared to skip the problems altogether.

Goal: CS PD is content-specific & job-embedded

CS PD materials: The materials were content-specific in the following ways: They addressed the mathematical content of the mission observed (fractions and they provided an overview of how fractions fit into the year's curriculum. At the problem level, the details of the content were

addressed and different representations for specific problems were shown and related to each other.

CS PD as observed: The enacted CS PD was job-embedded in several ways. The PD was just-in-time, conducted just before or while the curriculum "covered" fractions. It was highly correlated with the approaches to the topic used in the curriculum. It asked teachers to examine the curriculum and plan for how to use it. While there was no facilitator in most of the CS PD sessions and the CS PD videos did not directly address the teachers' practices or how to integrate Zearn Math strategies into their practices, half of the teachers talked about their own practices during the session. In the instructional interviews, six of seven teachers said the CS PD connected with their classroom practices. They all said the Zearn Math curriculum guided them in their teaching, and that the CS PD would be directly relevant to their classroom practice.

Goal: CS PD leverages peer teacher expertise and collective experience

CS PD materials: It's difficult to determine, from an analysis of the PD materials, whether teachers drew on each other's expertise and experience. This would be an important role for the facilitator, to encourage teachers to share their experience, and the facilitator's guide states that "Teachers will deepen their understanding by collaboratively examining the teacher materials, solving math problems using strategies from the mission, and analyzing example student work," which implies creating a collective experience.

CS PD as observed: Most teachers (six of seven observed) collaborated during the CS PD session; however, we noted that most of this happened naturally between teachers, without explicit prompting by the facilitator. We observed five of seven teachers sharing and discussing ideas, which seemed to work well and be productive. In two of the sessions observed, where a facilitator was using the intended guide, the facilitator was instrumental in generating the discussion. In one session, having others in the room appeared to slow the pace of the PD productively. (That is, having another teacher in the room prevented the teacher from just clicking through without working through the problems). But we also observed that some teachers spent considerable time in discussion between pauses in the video, which, while on topic, meant they had less time to either (1) complete the activity thoroughly or (2) watch the video entirely.

When asked in the instructional interviews about collaboration, most teachers said that the CS PD session supported their collaboration efforts. One teacher said the time of day impacted collaboration. Her PD session was at the end of the day, and she said that although the CS PD session facilitator fostered a lot of collaboration, she and her colleagues may have been too tired to collaborate effectively. One teacher said that if everyone is engaged in authentic problem solving, the collaboration was fruitful, but it was much less so if others don't engage in the same way. The only teacher who did not have someone in the room with whom to collaborate said that collaboration would be beneficial for him because he would have a partner

to bounce ideas off of; yet, he also said it was valuable to complete the CS PD session independently, as he could set his own pace.

Goal: CS PD helps teachers understand student thinking and develop responsive teaching moves

CS PD materials: Teachers are presented with frequent opportunities to think about and discuss student thinking, but there are limitations to how deeply teachers can explore this topic. Prompts to think about student thinking are definitely present. With “What representations of fractions will students use throughout this mission?” the question is more about representations than students’ thinking with them. However, teachers are also prompted to “Consider strategies students may use to solve” a problem for which no particular representation is specified. It appears teachers would need to answer this question based on their experience using the Zearn Math curriculum. In the videos we watched, students’ strategies were shown, but we did not see an incorrect solution presented. Teaching moves such as suggesting students use pictures or reminding students what a whole is are given without reference to what student thinking these moves may be in a responsive manner of teaching. This is key for considering whether teachers can develop responsive moves by using CS PD. Responsive teaching, in current research, includes attending to student thinking and responding with teaching moves drawn from an understanding of that thinking (Jacobs & Empson, 2016). These prompts move close to that standard but fail to meet it.

CS PD as observed: During most observations, the CS PD materials provided means for teachers to understand student thinking. The videos we observed showed only one method for solving a problem. While the videos overall showed different pictorial strategies, only one was applied to each problem type. The CS PD materials did prompt teachers to “consider strategies students may use to solve,” but most sessions did not use the participants’ guide as intended. One teacher noted, “[The CS PD] shows you about one way, so it does add something to your repertoire, but it is very concise. When the curriculum itself—if you go through each lesson, it shows you so much more." Additionally, as noted above, only about half of the teachers engaged in authentic problem-solving during video pauses. In one instance, the facilitator or other teachers in the room explicitly pointed out problems that students would likely struggle with and they discussed ways to address it. It should be noted that horizontal and vertical remediation was not addressed explicitly in discussion during any of the observed CS PD sessions. While methods of remediation are present in the participant’s guide, it is likely the topic wasn’t discussed because the participants were pressed for time and teachers read through the guides quickly.

In the instructional interviews, most teachers said the CS PD helped them understand student thinking: Three said it was because the PD overall demonstrates multiple pictorial strategies, and two said it was because the PD gave them prompts to push student thinking. Notably, one teacher said the CS PD did not help her understand student thinking because she gains that from observing what her students do while solving math problems.

Goal: CS PD supports strong lesson planning

CS PD materials: Lesson planning is prompted in the participants' guide. Teachers are asked to, for example, think of questions they may ask students to emphasize, say, fractions as numbers or to elicit the use of a representation. The facilitator would again have a strong role in emphasizing these opportunities for planning. We did not see a prompt for writing a whole lesson plan, which could be thought of as a requirement for strong planning.

CS PD as observed: We saw support for the lesson planning activity given in fewer than half the observations. Despite these opportunities being available in the CS PD materials, participants did not discuss lesson planning with each other. In some observations, our impressions were that the teachers intended to simply use the lesson materials directly provided by Zearn Math without much adaptation, so they didn't feel much of a need for lesson planning.

Teacher impressions of CS PD

We found that overall, seven of the eight teachers interviewed agreed that the CS PD sessions were helpful, but for varying reasons. (One teacher could not, remarkably, remember the PD session.) One teacher said the PD gave him insight into his own teaching because he "thought that gave [him] a way to look at [him]self just through another perspective to see what it looks like when [he's] teaching or what it sounds like when [he's] teaching..." Two teachers said the CS PD sessions were successful because they provided a good preview of the mission to come. One said, "I like the PD because it just prepares us for what we're getting ready to do, gives us an idea of all the questions that they're going to be asking." Another teacher said the PD sessions set the teachers up for success by providing them all the tools necessary to teach the mission successfully. Three teachers said that the chunking and sequencing of the videos made the PD sessions successful.

Most (five of seven) teachers said in instructional interviews that they found something challenging in the CS PD, including the following: too much information to grasp at once; inefficient use of time; questions asked did not generate good discussions (for example "questions that [are] like, 'What do you notice is the difference,' or those kinds of questions, sometimes, it's maybe obvious"); sessions were repetitive for teachers who watch the same video year after year; and there was not enough time to fully digest all the information presented. When asked how to improve the CS PD sessions, two teachers said they wished sessions were shorter or more concise. One teacher liked the PD sessions but was frustrated that her school did not provide her enough time to participate in-depth.

Teachers' Self Reports about Zearn Math Impacts

When we asked teachers about the impact of CS PD on their instruction, many described being influenced by "Zearn" more generally, including CS PD, the Zearn Math curriculum, and their experiences in implementing the curriculum.

Half the teachers said that Zearn Math has heavily influenced or somewhat changed the way they plan their lessons. Exact details varied: one teacher said that due to time constraints in math, she was able to use Zearn Math as a guide to know what the key skills are for students and to make sure she highlighted those in her lessons; one teacher said that while Zearn Math did not change her lesson planning, she realized through Zearn Math that it's okay to spend more time on foundational concepts because it will help her students more in the long run; one teacher mentioned that he used Zearn Math as a foundation, but modified the lesson to meet his students' needs. Three of the teachers said they couldn't give a "before-and-after" comparison because they were early on in their career and had only ever used Zearn Math in their teaching. These findings are puzzling, given that teachers appeared to lack the need for lesson planning.

Most teachers said they felt prepared to teach math to begin with, but also said that Zearn Math helped them improve their ability to teach math. They said Zearn Math gave them new ways to teach the subject and illuminated student thinking. One teacher said the CS PD content itself didn't help her, but that the collaboration during those sessions challenged her to think about math concepts and how to teach them differently. Further, most (six out of eight) teachers said their PCK (defined as how teachers learn to teach others or the understanding of how students solve problems) was strong, but also that there was also room to grow. They said that Zearn Math improved their PCK by showing them how to teach multiple methods for solving a problem.

Half the teachers said that Zearn Math helped them support students in understanding math at a deeper, conceptual level, in particular, because Zearn Math prompts students to show their work. When asked if Zearn Math influenced their knowledge of how students gain conceptual understanding, one teacher said that Zearn Math's emphasis on different methods to solve a problem helped students understand concepts differently. Another teacher said that Zearn Math's digital component made it possible for students to understand concepts in a kinetic, hands-on manner and a visual, computer-screen manner. One teacher said that Zearn Math's emphasis on conceptual understanding was a shift from a more traditional "drill-and-kill" method.

3.5.2 Cross-Case Findings - Teacher Implementation of the Zearn Math Curriculum

In this section, we characterize the case study teachers' implementation of the Zearn Math curriculum and how teachers described their experiences and approaches. These cross-case findings are organized by characteristics of the classroom according to the logic model: Teachers who have gained strong PCK through CS PD should: (1) create a classroom community where all students experience belonging; (2) foster rich student-led math discussions; (3) help students move from concrete to pictorial to abstract through problem-solving; (4) leverage Zearn Math to create a cohesive student learning experience; and (5) provide feedback on student thinking.

Goal: Create classroom community where all students experience belonging

All seven teachers who were observed and interviewed had some strong basics in place for building classroom community. They described their students in a positive manner, noting that they were capable and driven to learn. Most of them said they aim to create a safe classroom culture where students feel comfortable and confident to speak up and make mistakes. In most of the classrooms observed, there were no major behavioral issues, students were largely engaged, and teachers had classroom management strategies. Teachers were generally positive and there were indications of positive rapport and mutual respect between students and their teacher. All these qualities set the stage for student experiencing a sense of belonging. We did not, however, note other aspects of supporting a sense of belonging for students who are historically marginalized, such as providing encouragement along with critical feedback on students' work (Romero, 2018). There is no reason to believe that these were not part of teachers' practice; we simply didn't observe them on our visit.

Goal: Foster rich student-led math discussions

In instructional interviews, teachers said they try to elicit student thinking and engagement during discussions by prompting students to share their thinking publicly and in small groups or pairs. Most teachers said their students are generally engaged; however, three of them also said that engagement varies by achievement level, with higher-achieving students speaking out more than struggling students.

In practice, only two teachers truly elicited student reasoning by building on and involving other students in more elaborate student-focused reasoning. Especially, one teacher would consistently ask students how they arrived at their answers.

The logic model describes one aspect of students' experience as having agency. Human agency has various definitions in different disciplines, but mathematics educators have converged on a fairly constant interpretation: When students have agency, they engage in purposeful activity, creating new ideas or extending established ones (Sengupta-Irving, 2015).

The case study teachers had a somewhat different view of agency: they seemed to equate student agency with students' ability to self-pace in the digital component of Zearn Math. All teachers said students have considerable agency and one teacher went so far as to say students might have too much agency over their learning.

While teachers reported that students had agency, our lesson observations revealed fewer opportunities for students to create or build on ideas for themselves. Seven teachers' instructional approaches could be characterized as teacher-centered. Two teachers exclusively used the IRE (initiate, respond, evaluate) approach (Mehan, 1979). With this approach, students' opportunity to participate is limited to a short answer which the teacher immediately evaluates. Two teachers balanced the IRE approach with teaching that built on student input and responses in their "initiations." In one class, the lesson was teacher-focused during the

whole-class lecture and discussion but more student-focused when students worked individually. The lesson began with her own demonstration of how to solve a problem, but then the students' ideas and initiatives shaped the rest of the lesson. Another teacher's class was student focused and also very adaptive; students drove the discussion and the teacher adapted to their pace, enabling agency as contributing ideas as well as setting one's own pace.

A question related to agency is, who is doing most of the cognitive work (Henningsen & Stein, 1997) in classroom discussions. In the lessons we observed, six of the teachers did the cognitive work, providing the reasoning for solving a problem, with some student input. However, two teachers allowed students to drive the questions, conversation, and reasoning. One teacher collaborated with students to problem solve and explain; initially, the teacher scaffolded a lot of the conversation and did the cognitive work but then reduced support to be more student-driven as the class progressed.

Goal: Help students move from concrete to pictorial to abstract through problem-solving

Helping students move from concrete to pictorial to abstract includes guiding students in problem-solving initiated through visualization and conceptual exploration grounded in the concrete. All teachers used some form of pictorial method to explain abstract concepts in the lessons we observed, while over half of the teachers did not use concrete representations of fractions. (We only observed one lesson for each teacher, and given that the concrete to pictorial to abstract spectrum might unfold over more than one lesson, teachers may have covered other aspects in prior or subsequent lessons.) Their pictorial strategies ranged, but many teachers used some form of a model from the Zearn Math curriculum: number lines, tape diagrams, student-created visuals, or area models. One teacher's use of the pictorial was quite limited; she only used the number line method once. The teachers who incorporated more concrete models used methods such as using a concrete scenario of a specific student in the class going to the store, using physical blocks to represent fractions, and using an analogy of the class sharing pizzas to explain wholes and fractions.

Goal: Leverage Zearn Math to create student cohesive learning experience

The Zearn Math model includes the use of a digital component as well as students solving problems in small groups and with the teacher; creating a coherent experience means at least in part coordinating these three modes. Teachers varied in how much time they devoted to each. About half of the teachers observed assigned half of the lesson time to the Zearn Math digital component and half to whole-class and small-group classroom instruction. Two teachers spent approximately two-thirds of lesson time on classroom instruction and one third on the Zearn Math digital component. In one teacher's lesson, students spent almost the entire time on the digital component, with just a small group receiving regular instruction. Most teachers (five of six) said there was a good balance between the digital and hands-on components of Zearn Math. About half of the teachers said that even though there was a good balance between the

two, they would have preferred more time to work with students in person. One teacher said some activities were too difficult to do digitally, citing the geometry unit with protractors as an example.

When making connections across grade-levels, another aspect of cohesion, over half of the teachers said they reference skills they know students learned from previous years when discussing math. Oftentimes, they would directly refer to skills students learned from Zearn Math the year before and remind them of it. Two teachers began the observed lesson by reviewing prior skills students needed for the new lesson.

Remediation is a part of the Zearn Math model. In addition to the embedded remediation in the digital component of Zearn Math, half the teachers said they remediate by pulling aside a small group of students that may be struggling and dedicate more time to them. Two teachers said they use Zearn Math data and exit tickets to identify which students are struggling and where. All but one of the teachers remediated by walking around the classroom visiting individual students one-on-one when they were struggling through a problem. In addition to individual remediation, one teacher would also address the whole class if she saw multiple students struggling with the same concept. Half the teachers said they had not learned anything new or different in how to approach remediation through Zearn Math CS PD or lessons. However, notably, one teacher said the Zearn Math CS PD and the Tower of Power helped him with his approach to remediation because he said he can see where students are struggling and discern where he should go back to remediate.

Goal: Provide feedback on student thinking

In small group work and as students worked on the digital component, the teachers showed evidence of giving precise, timely, and safe feedback. All but one teacher offered immediate feedback to individual students as they worked through problems. They would offer positive reinforcement when students solved a problem correctly and would point out what a student got wrong, encouraging them to try again. In contrast, they did not typically elicit student thinking to help them understand why the student got it wrong. Teachers would circulate the room as students worked on the digital component of Zearn Math, helping students when they got stuck. It was difficult to tell what feedback students were receiving directly from the digital interface. Notably, one teacher encouraged students to work through problems themselves and not ask the teacher for help during the digital component.

3.5.3 Within-Case Study Findings

In this section, we describe the PCK findings for each of the eight case study teachers and the hypothetical explanations for the relationships between teachers' PCK, Zearn Math experiences, and instructional approaches.

Phillida

Phillida had an elementary education certificate and taught third-grade math for two years at San Lorenzo Elementary. The other Grade 3–5 teachers at her school had similar teaching experiences.

Phillida demonstrated moderately strong PCK. She made no mathematical errors, despite her expressed discomfort with fractions. Her teaching strategies were most tightly tied to specific representations, with some growth between the interviews in how much her choices would be based on students' thinking.

For example, she noted that for students who were having trouble understanding why 5/6 > 5/7, she would have students act this out, with groups of students (one, two, three..) coming to the front of the room and ask, how much pizza does each person in this group get? She believed students had realized that the number in the denominator indicates the size of the pieces. This teaching strategy was tied both to a representation and to student thinking because she said she would use it when she identified students who were having trouble with the concept during her instruction.

In both interviews, she cited Zearn Math—as a whole—as a main source of her development of teaching strategies. She said she studied the material carefully because fractions were hard for her and her students.

A combination of self-perceived weakness in math, buy-in and commitment to the Zearn Math materials and PD, and reliance on Zearn Math materials and scripts may explain Phillida's moderately strong PCK. Phillida was in her second year of teaching elementary school. She expressed that math is not her "strong suit" and that she had difficulties with fractions. Despite this stated lack of confidence in her knowledge of fractions, she demonstrated moderately strong PCK in both her PCK interviews, with more strength in her knowledge of how representations could be used to solve problems than in other areas of PCK. Some of this PCK seemed to be evident in the observed lesson, as when she guided students through representations to be used in solving problems. In her instructional interview, she stressed her reliance on the "scripts" that Zearn Math provides in the teacher materials, perhaps because of her lack of confidence. In her instructional interview, she said the most important classroom practice was being familiar with the materials such as the lesson, the standards, and the digital components. This might be why she studied the Zearn Math material carefully. She was also very engaged during her PD session which could indicate her strong willingness to deeply understand the concepts that she teaches to students. The scripts may have also been a source of learning. In the observed lesson, she was better able to support students' correct answers (asking them to explain their answers) than the incorrect responses, where she asked students to think again, a very general response, or simply asked another student to respond. This may be because this knowledge from practice was gained after the observation. Additionally, Phillida

paid attention to student affect by offering plentiful praise for correct answers and using growth mindset materials.

Britt

Britt taught third grade at Fullerton and was in her first year of teaching. Prior to this year of teaching, she was a math classroom aide at her current school.

Britt's PCK was moderate. She made some mathematical mistakes in the first interview and left out the representation that had been problematic (number lines) in the second. Her teaching strategies were related to the representations used in Zearn Math—a strategy for each representation that she mentioned. However, she did not describe teaching strategies based on student thinking. She did not specifically address giving students choices—or not—in the representations she might use.

For example, to show that 5/6 > 5/7 she said she would draw, or have students draw, two equal-sized squares and divide them into sixths and sevenths, the denominators of the two fractions. Her descriptions of how to help indicated she would provide the models, pictorial and physical, to students, and guide their thinking.

Britt cited Zearn Math as a source of learning in her second interview but not in the first, indicating Zearn Math as a new source of learning for her. Britt would benefit from more PD on the "transitional" representations: number lines and number bonds and a chance to learn more about student thinking

Britt's PCK was limited in her attention to student thinking and notion of student agency in problem-solving, which was mirrored in her classroom. Her instructional approach was teacher-centered and relied on behaviorist approaches and the notion of the teacher as the intellectual authority (Amit & Fried, 2005). It seems her understanding of math instruction was more focused on procedure than the complexities of student problem solving and reasoning. She did not appear very engaged with the CS PD and showed less insight into student thinking compared to other case study teachers. She said she was good at math as a student and felt prepared to teach it; but it's possible her own math education was focused on rote learning, which has long remained most common in American classrooms (Wood et al., 2006), and her sense of preparation may well have been likewise limited to the more technical aspects of implementing curriculum and informed by her own experience as a math student (Ball, 1988). Britt was a very new teacher with no elementary school degree and so had no formal training in methods. She was possibly not disposed to think critically about new information and skills which also might be attributed to her lack of training in formal methods (Darling-Hammond, 2000).

Juanita

Juanita taught fourth grade at Fullerton and had an elementary education certification. She had been teaching for four years and using Zearn Math for three.

Juanita's PCK was rated as strong. She was confident in the mathematics of fractions, of common student preconceptions, and of the methods of problem-solving her students tend to be comfortable with. In the second interview, she discussed more about students' conceptions behind errors, whereas, in the first interview, her focus was more on correcting the error.

For example, in the second interview, she explained that through a student's reasoning about the problem 5 – 4 ¾, she could find out if a student did not have a conception of fractions at all and go back to very basic ideas and build from there.

In both interviews, Juanita cited Zearn Math—as a whole—as one source for her teaching strategies, but also attributed Eureka Math, which she saw as the main curriculum she used. These two are related, in that they are both based on the curriculum first distributed as EngageNY. As such, they share, for example, a focus on the progression from concrete to pictorial to abstract. CS PD could help her gain even more confidence in dealing with the early conceptions of fractions.

Juanita had strong PCK and was particularly attentive to strategies focused on student thinking and understanding. Though she said that she struggled with math her first year of teaching, Juanita, now in her fourth year of teaching, had created a vibrant, student-driven math class engaged in discussion that had students building upon each other's ideas. She was a playful teacher who invited students to debate her. She also said she saw differentiation and remediation as important foci of her instruction. She had used Zearn Math for 3 years and said that it helped her understand math concepts in a way that allowed her to identify student misconceptions. She also attributed Zearn Math with helping her overcome her dislike and fear of math. In her PCK interview and classroom observation, Juanita showed that a strength of hers is adapting her strategies and instruction based upon each student's needs. This picture of her teaching is consistent with her strong PCK in which she was able not to discuss common student errors and how she would help them correct them, but what the error indicated about the student's conceptual understanding.

Jammie

Jammie taught fourth grade at Laurel and has been teaching for 12 years. She is one of two case study teachers with over 10 years of experience teaching.

Jammie showed growth in PCK over the two interviews, from moderate to moderately strong. She correctly described a student error as presented in the interview materials; however, in the second interview, she was able to say more about possible preconceptions behind the error. In the second interview, she explained using representations in an instructional sequence, rather than simply citing an instructional strategy for each representation.

For example, she talked about a range of strategies she would use, from visual representations to number lines and equations, and also how she could help students move from one to the other, culminating in the symbolic.

Jammie's description of where she learned each strategy she described shifted from the first interview to the second. She stressed her prior teaching experience in the first interview although she mentioned Zearn Math as well. In the second interview, in discussing where she had learned her teaching strategy, she had much more to say about what she learned from Zearn Math; she cited Zearn Math, as a whole, and Curriculum Study PD in particular—as well as her prior experiences. CS PD could help her further understand how students' thinking develops.

It seems Jammie benefitted both from the curriculum and the CS PD. She was a very experienced teacher, who said she valued conceptual learning, and who seemed to embrace the Zearn Math curriculum and ideas. She was familiar with big ideas and worked actively with them in planning and implementation. She said that Zearn Math had helped her with learning new and different strategies to respond to different types of learners. She said that Zearn Math made student thinking visible, which helped her understand it better. In CS PD, we saw a lot of peer discussion, interaction, discussion of past and current students and practices. The CS PD facilitator complemented the videos in ways that supported understanding of student thinking. At the same time, Jammie's PCK was not strong, but moderately strong. Her classroom seemed to be characterized by traditional classroom management approaches with a fair bit of teacher control mixed with some student choice. Her description of her instruction was more student-focused than what we observed in her classroom. It's possible that her PCK was limited by a more traditional, teacher-centered paradigm in the classroom.

Demetris

Demetris taught fourth grade at Laurel and had an elementary education certificate. She had been teaching for three years. This was the first year she taught departmentalized math.

Demetris demonstrated strong PCK. She made no mathematical errors in the interviews. She showed growth between the two interviews in both her ability to diagnose student errors and how to help students develop increasingly sophisticated understanding of concepts. She confidently discussed multiple teaching strategies she could draw on, even describing detailed sequences of questions she would use.

There was a marked difference in her attribution to where she got her methods: in the pre-interview, she indicated a number of sources, mostly based on her experience, but she also cited Zearn Math. In the post-interview, she attributed only Zearn Math, saying she would have taught the topics completely differently before Zearn Math (in her estimation: 75% teacher guide, 35% other Zearn Math resources, including PD but did not call out CS PD). CS PD could help her further coordinate her teaching strategies.

For example, she said that before she taught Zearn Math, to solve 5 – 4 ¾, she would have had students rewrite the mixed number as a fraction, and the whole number as a fraction with the same denominator, then use the algorithm to subtract. In contrast, in both interviews, she explained how she could use number bonds to help "break up" the numbers.

Even though Demetris was a relatively new teacher, and, like many other teachers in the sample, indicated some behaviorist assumptions about students and classroom practices, she seemed to be a very student-oriented teacher, curious and interested in her students, and intentional about understanding their thinking, which could explain both her existing PCK knowledge and the additional learning from pre to post. She emphasized making a connection to students, making math relatable. She said she approached discussions by having students teach her content, and then as they're walking her through steps if there are misunderstandings, other students help each other out. She said when she planned, she modified the lesson to fit her students, making sure she could reach them. Given that she was a teacher that appeared to want to understand her students better, the Zearn Math curriculum's focus on student misconceptions, conceptual learning, and strategies for identifying and working with student errors and struggles were meaningful to her. During the CS PD session, Demetris was engaged in discussing and collaborating with other teachers and worked on solving problems authentically. She used multiple strategies to solve each question independently and offered up the different ways she solved them during the discussions. During discussions, she also used Zearn Math terminology. Her PD session had a strong facilitator, which she thought was too rigid and didn't allow her and her colleagues to delve deeply into the content. She said she would go back and re-do the PD at her own pace. Her dedication to understanding Zearn Math materials, and the way she leveraged collaboration could explain her strong growth between PCK interviews in her ability to diagnose student errors and how to help students develop an increasingly sophisticated understanding of concepts.

Jard

Jard taught fourth grade and had been teaching for two years at San Lorenzo. Jard had an elementary certificate as well as a special education endorsement.

Jard's PCK was rated moderate for the first PCK interview and moderately strong for the second interview, indicating growth. He showed some weakness in content knowledge. Thus, he showed growth both in teaching strategies based on representations, as well as a willingness to work with students' conceptions in instruction.

For example, in the first interview, he cursorily described his teaching methods, which came across as procedural. The second interview revealed that what he had described as teaching through "modeling" in fact included time for students to solve a problem on their own and present their solution, rather than simply demonstrating a process that students can follow, as "modeling" often implies.

He cited CS PD specifically when asked where he learned his teaching methods, in the second interview. He did, however, say that he thought the Zearn Math CS PD focused only on correct reasoning, and he would like to see more presentations of students' misconceptions and how to help them. He also cited his mother, who is a teacher, as helping him learn to teach. He noted, too, his own experiences in school, where he liked to use his own methods, different than those

the teacher presented. CS PD could help him focus on student thinking and early conceptions of fractions.

Jard demonstrated the most notable shift in PCK knowledge from pre to post, but we're not sure if this reflects actual PCK growth or the fact that he spoke in more detail in the second interview. One strength was how he described instruction that allowed for students' own thinking, as well as responding to that thinking. On the other hand, he seemed unable to follow student reasoning in the video and relied on the standard algorithm as the way to solve the problem the student was presented with. In the observation and instructional interview, Jard presented a bit of a puzzling contrast. On the one hand, he seemed to be a dedicated and engaged teacher, who valued student thinking and independence, and cared for math as a subject. He was a new teacher, whose favorite subject in college was math, whose mother was a teacher and who he said taught him how to teach, and who purported to like using his own methods. He also asked students for their reasoning during discussion and tried to build on student ideas. He was highly engaged in CS PD, thought it impacted his teaching, showed him different ways for students to solve problems, and said he had incorporated strategies into his own practice. On the other hand, he took a very traditional approach to instruction, focused on the lesson script, and didn't demonstrate a strong sense of student agency or thinking. While Jard talked about wanting to use his own methods as a kid, and for his students, and used some of his own methods in CS PD, he also emphasized following the script when it came to the Zearn Math curriculum, both in planning, implementing, and what he considered to be important in Zearn Math. He did not like the digital aspect, saying it was difficult for planning and for students' conceptual learning, but also seemed to cede control to Zearn Math digital (e.g., for across grade-level planning or referring students to Zearn Math remediation when they had questions). When asked about student agency, he said students have agency when they use the computer. During observations, while asking students for elaboration, he also mostly was the one driving discussion and doing the reasoning with fill-in-the-blank student participation. He emphasized the focus on math vocabulary and seemed to have less to say about student thinking and conceptual struggle. If we were to say that Jard's PCK increased, it may have been influenced by CS PD, which he was highly engaged in, including solving student problems, and he had a good foundation to build on as someone who was interested in math and had a family background in teaching. He said he thought it had impacted his teaching, that it showed him different ways for students to solve problems, and that he's incorporated strategies into his own practice.

Diana

Diana taught fifth grade at San Lorenzo and had been teaching for 14 years. She was one of two teachers in the study with over 10 years of teaching experience. She had an elementary education certification as well as a special education endorsement. This was Diana's first year teaching general education. Previously, she taught special education.

In both interviews, Diana displayed strengths in her intertwined descriptions of where students' difficulties may lie and how her teaching strategies were based on these. This intertwining of knowledge of student thinking and teaching strategies indicates very strong PCK. She also seemed confident in her ability to work with whatever strategy students were using, and said she encouraged her students to be "free thinkers." She described in both interviews an instructional sequence to move students from direct modeling to writing number sentences. Her articulation of this sequence was detailed and explicit in the second interview, but more implicit in the first.

For example, in response to students' difficulties, she said she would help them understand fractions as numbers by using visuals such as fraction strips to show how denominator and numerator relate, then use benchmark fractions and help students understand the whole's special role, and finally use anchor charts to help students remember how to use a number sentence.

Diana said that before Zearn Math, all she knew to teach was how to do the algorithm, so she had clearly learned much of what she shared from Zearn Math—she did not describe the effect of any one component of Zearn Math on her practice, except to note that fraction strips were from Zearn Math. CS PD could help her find ways to push student thinking through additional questioning techniques.

Diana's PCK was strong in both interviews, and she was able to identify students' difficulties as well as adjust her teaching strategies based on specific difficulties. In her instructional interviews, Diana said that getting to know her students and anticipating their misunderstandings were key to her practice. She said that it was not through Zearn Math, but rather directly working with her students, that she understood student thinking. She did say that Zearn Math pushed her to go more in-depth with her students through questioning, and the classroom observer noted that her feedback to students occasionally came in the form of questions when students answered questions incorrectly. At the same time, the observer in her classroom found that she made considerable use of the IRE format and did most of the reasoning. It could be that her strong PCK came from her many years of teaching experience, along with her belief that students should drive learning. She was not very engaged during the CS PD session, which may explain some inconsistencies in her assessment of the usefulness of Zearn Math materials. She also mentioned that she wished the PD was much shorter, perhaps because she had strong PCK and only found the student perspective and the peer collaboration pieces useful.

Will

Will taught fifth grade at Fullerton and had an elementary education certificate. He was in his second year of teaching and had used Zearn Math for two years.

Will consistently demonstrated weak PCK in both interviews. While he was confident in his knowledge of the standard algorithm and vocabulary associated with it, he did not show an

understanding of fractions that underlie this algorithm. Similarly, his teaching strategies were completely reliant on the standard algorithm, as well as leading students through the steps to get a correct answer, with no description of strategies to support students' understanding of fractions. Will's description of his practice in the second interview was almost identical to the first interview.

For example, he stressed the importance of expressing fractions in lowest terms, without referring to helping students understand the meaning of this equivalence.

It should be noted that Will demonstrated a very caring attitude, in the way he spoke of students' experiences in his class and their lives outside the classroom.

Because of this consistency and focus on procedures and lack of use of multiple representations, it seemed he had learned little from Zearn Math, be it PD or curriculum. If used under the right conditions, CS PD could help Will uncover and expand on his conceptual knowledge of fractions.

Will's PCK was weak and procedural and did not change over time, likely because he was a new teacher with a very traditional, IRE approach, with accompanying procedural beliefs about math teaching and learning, which were reflected in his comments and teaching practices. He indicated on several instances that he had a preference for procedural approaches, that math was about teaching techniques or methods. While he expressed appreciation for the fact that students may learn more conceptually with, for example, the tape diagrams, he suggested there were more efficient ways. He also preferred math on paper instead of the computer. He was engaged in the CS PD and seemed to have a good relationship with students, at the same time that he expressed a superficial idea about student agency. Will's disposition seemed to be that he could get by using old approaches and thinking, and there was little in the curriculum, PD, or classroom implementation that necessitated a different frame of mind. At the same time, this confirms something we already know—that it's difficult to change teacher beliefs and practices (Kennedy, 2005). A more illuminating explanation might be that the Zearn Math curriculum may not necessitate teachers shifting old thinking and practice enough; i.e., teachers can get by with students doing Zearn Math on the computer and supplementing with regular classroom practice and support. In fact, Will said he was fine with students choosing on their own if the Zearn Math approach worked for them, and, if not, they could do something more procedural. Curriculum Study PD might focus well on the "how" but perhaps not enough on the "why," i.e., making a persuasive argument for teachers to challenge their existing thinking and practice.

We also note that although Will, in the PCK interviews, stuck firmly to discussing standard algorithms and procedures such as expressing fractions in lowest terms and did not talk about multiple strategies for helping students with misconceptions, in his teaching, he used multiple representations to support students' understanding. This was consistent, too, with what he described in his other interviews. Will described himself as "still learning" and this could explain these differences: his learning was at a point where there was a mix of ideas and practices.

Tape diagrams are a good example of a representation that can support understanding, which he used in his teaching, but did not discuss in his PCK interview. Also, though it was noted in his classroom observation that "When students contributed, it was mostly evaluating or confirming their accuracy/restating". It's important to note that Will talked about and demonstrated a very caring attitude toward his students as people, not just learners, with lives outside the classroom that affect what they do inside the classroom, which has increasingly been identified as an important teacher disposition and practice for historically marginalized students (Battey, Neal & Hunsdon 2018).

4. Discussion & Recommendations

In addition to finding that six out of the eight case-study teachers improved in assessed PCK between pre-and post-interviews, an overall finding of our study was that almost all teachers cited Zearn Math and its multiple paths for professional learning as a source for developing new understandings of representations of fractions and development of new pedagogical strategies. Some of that learning happened outside of PD, as evidenced by teachers' knowledge of these elicited during the first interview; however, almost all teachers did demonstrate growth in PCK between the two interviews. The multiple sources of learning cited included the activity of teaching, the student and teacher materials, the online learning component of Zearn Math for students, other teachers, CS PD, and other PD that Zearn Math provides. Additional non-Zearn Math sources were cited, too—prior teaching experience, family traditions, PD from other providers, and supplementary curriculum sources. That these teachers learned from these multiple sources is not surprising and is supported by the literature on teacher learning (Davis & Krajcik, 2005). Though this study was initially proposed as a study of the effects of CS PD, early data indicated the question should be broadened to adequately take into account the evidence we were gathering. Additionally, due to unavoidable delays in scheduling, the first and second interviews did not precisely pinpoint CS PD. However, even if they had, it would not have been prudent to ignore what the initial data were revealing.

The categories of PCK examined have been described earlier in this report. It is worth noting, though, the two kinds of pedagogical strategies we identified: strategies related to *representations of fractions* and strategies related to *students' thinking*. The teachers in this study overall described more strategies based on representations and somewhat fewer about students' thinking. There are multiple possible reasons, but one teacher noted that CS PD focuses more on correct reasoning than incorrect reasoning, and in our review of CS PD materials, we noted the strong emphasis on multiple-representations strategies. However, because few teachers cited CS PD alone as a site of learning, this is not conclusive evidence that CS PD contributed to this imbalance.

The imbalance is important to note, though. First, because Zearn Math is designed for teachers to give safe, in-the-moment feedback that precisely addresses areas of misconception. Second, because a long line of research on learning and teaching math reveals that a focus on students'

thinking is of great importance (Jacobs et al., 2010; Richards & Robertson, 2015). A prime example is the program of research, Cognitively Guided Instruction (CGI) (Carpenter et al., 1996), which came to this finding in the late 1980s. Researchers relying on CGI have continued uncovering patterns in student reasoning and portraying them to teachers, in the areas of whole numbers, fractions, and decimals (Empson & Levi, 2015). **Thus, one recommendation for Zearn Math is to follow this lead and include in CS PD an additional focus on students' thinking in PD and teacher materials, with particular emphasis on early conceptions and how teachers can support students in developing more sophisticated reasoning.** CS PD does address student thinking, but a review of the materials confirmed that there is less focus on how students may be misunderstanding aspects of fractions than on varieties of correct answers. In addition, there is little advice on or activities for learning how to respond to individual students' thinking as it unfolds, which would be important in differentiating instruction as well as providing timely feedback.

Frameworks for PCK do not typically include categories of providing students with agency or at least choice in mathematics, nor for caring about and for students. But recent, broader views of teaching indicate the importance of these qualities, particularly for educational equity purposes (Aguirre, et al, 2013). We have noted evidence of these in the case write-ups. With regard to student choice, typically teachers first demonstrated or asked students to use a particular representation, while providing an opportunity for choice in representations in culminating problems, consistent with the curriculum and the sequence of problems in CS PD. One teacher went a bit beyond that, saying she would encourage students to be "free thinkers," which implied greater agency, as defined earlier in this report as a purposeful activity, creating new ideas or extending established ones (Sengupta-Irving, 2016). With regard to caring, some teachers also spoke of students' feelings about mathematics learning, and one teacher in particular spoke about students' lives outside the classroom. Both of these are consistent with recent research on teacher caring (Maloney & Matthews, 2020) and with the Zearn Math logic model. **Broadening the focus of CS PD to include student agency and teacher caring could have an influence on how teachers implement Zearn Math.**

A further recommendation is to differentiate the PD. Based on the PCK interviews, we made a recommendation for each teacher on what they could learn from CS PD, based on the three categories of PCK we attended to: mathematics content; representations as content and as the basis for teaching strategies; and teaching strategies based on student thinking. Our recommendations: conceptual understanding of fractions for one teacher, content knowledge and teaching strategies based on a representation of fractions for one teacher, and we thought four teachers could be helped with strategies based on student thinking. CS PD could be differentiated along these lines. In our experience, differentiating teacher PD is challenging, but could be worth it to meet these differing needs.

Finally, the tensions we saw across the cases between implementing Zearn Math's student-oriented instructional practices and the features of traditional classroom structures are similar to

those seen in numerous studies over the decades (e.g., Cohen, 1990; Kennedy, 2005), and remain common today. We know that classrooms around the country have not all made the shift towards positioning students as more active sense-makers, and that the majority of instructional time in math is structured around whole-class activities that generally do not afford students opportunities to deeply engage with math and science content and practices (Banilower et al., 2018). The Common Core State Standards in Math (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) emphasize student learning of practices, problem solving and sensemaking, constructing and critiquing arguments, strategic use of tools, and modeling. Many school districts are still in the midst of wrestling with curriculum adoption, aligned assessments, and appropriate professional development to support teachers in enacting instruction that reflects these standards and visions (Polikoff, 2015).

Professional development has typically served as the primary avenue for supporting teachers in their learning and implementation of new instructional practices. PD has the potential to influence teachers' knowledge, practice, and views (Cohen & Hill, 2001; Garet, et al., 2001). But teachers also need instructional resources that scaffold their emergent views and understanding of reform, foster changes in their instructional practices required by reform, and make connections between the goals of the reform and states' accountability expectations. Given our findings in this area, we suggest, in addition to our recommendations related to CS PD, that **Zearn consider additional ways for teachers to engage in conversations and activities focused on how to incorporate student-oriented instructional changes in the context of their beliefs, values, and instructional contexts**. Through existing or new forms of PD, Zearn may provide teachers opportunities to test and reflect on practices that are new to them (Clements & Sarama, 2016), framed within a continuous improvement mindset (Vescio et al., 2008).

5. Appendices

Appendix A - Teacher Case Descriptions

Phillida

Phillida had an elementary education certificate and taught third-grade math for two years at San Lorenzo Elementary. The other Grade 3—5 teachers at her school had similar teaching experiences.

CS PD Experiences

Phillida and her colleagues were highly engaged with the CS PD activities and each other, collaborating, asking each other questions, sharing strategies, and connecting ideas to her students. When working through problems, Phillida solved them aloud with colleagues. She seemed excited to incorporate new strategies into her practice. She appeared to want to

understand students' thinking to support their math learning. She also consistently used Zearn Math's vocabulary when discussing problems, strategies, and content with colleagues. In her instructional interview, Phillida said the videos were helpful because they provided examples and showed how to solve problems. In particular, she said they helped guide her to push student thinking.

Classroom Observation

The lesson we observed was focused on identifying equivalent fractions. It was structured around an application problem that used visual representations, followed by students working in small groups. Most of the lesson was grounded in concrete and pictorial representations of fractions. In the main lesson, Phillida used blocks of different colors to visually represent fractions and rearranged them to show that whole parts can take different shapes while fractional amounts remain the same.

Phillida created a positive classroom environment where students were respectful of each other; however, we also observed some overlooked potential opportunities for addressing students' struggles. During math discussions, when students had trouble figuring out how to solve the application problem, she encouraged students to listen to each other; however, she provided little feedback in pushing their thinking or guiding them towards a strategy that would help them. When students were incorrect, she didn't have other students or herself clarify. When they were correct, she elaborated on their responses. Few students participated actively in whole class discussions. The students and Phillida had an established call-and-response routine that she used when the students needed to answer a question or there were disruptions. Phillida primarily asked students fill-in-the-blank (Wood, 1998) or yes/no questions. Occasionally, she elicited student reasoning, such as when she asked a student to compare the similarity between two concrete representations of fractions and explain her answer. Her responses were mostly evaluative (e.g., "good" or "not quite, can someone else answer?"). Phillida drove mathematical reasoning with some student input during the small-group lesson. She mostly moved through the lesson as scripted.

Overall, this was a teacher-focused classroom. Teacher reasoning and direct instruction characterized the lesson and teacher questions to students took the form of recitation with mostly closed and a few open-ended questions. Students had a few chances to speak among each other to discuss answers to problems during the lesson and the teacher encouraged them to use "math talk" when they were discussing.

Teacher Perspectives on Instruction

Phillida said that to plan her lessons, she reviewed the Embark videos and read over the Zearn Math materials. In particular, she focused on the lesson script and student debrief to review the questions that she would be asking students. For remediation, she used lessons from the previous grade and pulled small groups to reinforce specific skills students may still be

struggling with. When describing her approach to math discussions, she said she would link problems to real world experiences. She said she was certain students were engaged because they used their whole brain and she asked them to talk to their partners. She said she believed students had agency over their learning with the Zearn Math curriculum. To make connections between and across grade-levels, Phillida said she referred to skills students learned the year before, a practice she didn't attribute directly to Zearn Math, while she also said that Zearn Math's digital component reiterated skills that were taught in the prior year. She said she had not learned anything about remediation from Zearn Math. However, she said she learned foundational concepts through the CS PD sessions, which was particularly helpful as she did not consider herself to be strong in math. She also said her knowledge of how students learn changed with the Zearn Math curriculum and CS PD session. The multiple strategies that Zearn Math provides allowed her to see how students think as well as allow her to adjust her strategies based on her students' needs.

Britt

Britt taught third grade at Fullerton and was in her first year of teaching. Prior to this year of teaching, she was a math classroom aide at her current school.

CS PD Experience

Britt was moderately engaged during the PD session, in the sense that she completed all the activities and collaborated with her grade-level partner; however, she did not seem particularly enthusiastic. We observed that she and her partner would often skip to Zearn Math's answer instead of authentically solving problems themselves.

Britt and her partner collaborated, but this seemed to be because of their pre-existing relationship and not necessarily due to Zearn Math's prompting. They appeared to be close colleagues, they did not have a personal facilitator, and we did not see any specific prompts for collaboration in the CS PD video. Their collaboration worked well because Britt appeared to bring stronger content knowledge while her colleague appeared to have a deeper understanding of the students.

During her instructional interview, Britt did not recall the CS PD session, which might be because the school uses both Zearn Math and Eureka, which she referred to as "all that other EngageNY stuff." In general, Britt mentioned that she liked how Zearn Math shows a lot of different strategies—particularly visual strategies—to solving problems.

Classroom Observation

The observer's impression of Britt's classroom atmosphere was that it was calm and positive, and students appeared engaged in the work. There was not, however, much room for discussion, as students mostly solved problems independently and shared out to the rest of the class. Britt had a behaviorist approach to classroom management (relying on rewards and punishments) (Freiberg & Lamb, 2009). For example, she displayed a chart with stars and

clothespins with students' names on it. Students started at five stars and worked their way down depending on negative behavior. She put smiley faces on correct student work and praised students for correct answers. In her instructional interview, she said she uses a lot of praise, in particular concerning students' exit ticket results.

Britt guided student problem-solving by first demonstrating an example and then having students do it on their own. She emphasized visual methods of problem-solving. When students worked through problems on their own, she provided guidance when students needed assistance. In these instances, she verbalized her own reasoning rather than ask students to elaborate or clarify their thinking. While students actively participated in solving problems, they did not discuss errors or solutions but rather appeared to work through the steps Britt had modeled.

Teacher Perspectives on Instruction

Britt said she relies on student repetition and following the 'I do/you do/we do' model, also known as the gradual release of responsibility model (Webb et al., 2019). She said she sees herself as a visual learner and that her teaching style relies on visual tools. In the observed lesson, she frequently focused on the pictorial aspects of the fractions--they did number bonds, number lines, and area models to solve the problems.

Britt's description of her lesson preparation indicated a teacher-as-authority role. She said, "Well, they encourage us to think about what they might struggle with, so I just always have an answer for it before they get into it, so I know how to answer it," which suggests less attention to facilitating student conceptual struggle.

Britt said she felt well prepared to teach math because math was her favorite subject growing up and she was good at it. As a new teacher, she relied on the Zearn Math curriculum, following the lesson plans closely; but she said she didn't know if Zearn Math had helped her feel more prepared. She did, however, say that the curriculum taught her new ways of solving problems and increased her knowledge of how students solve problems. Her language suggested some unease with these new methods, however: "I think the way we're learning math right now is different from how we learned, so I'm kind of trying to—I don't even know the word I'm looking for—conform?" When asked about her insights into students using the concrete-visual-abstract progressions, she said, "I honestly don't really see it, because I'm teaching while they're on Zearn. It's more of my TA that's moving (around), even though she wasn't there yesterday. She's more seeing that than I am..."

Juanita

Juanita taught fourth grade at Fullerton and had an elementary education certification. She had been teaching for four years and using Zearn Math for three.

CS PD Experience

We did not observe Juanita's CS PD session. However, since Juanita had used Zearn Math for three years and participated in CS PD sessions for each mission three times, she had considerable experiences to draw from in her instructional interview. She said she completed the fractions-focused CS PD with her grade-level partner who she said was new to teaching. Juanita said she liked how each section of the lesson was broken down into parts that built on each other, and that the PD showed teachers the prior knowledge that students have going into the lesson. She said the CS PD taught her everything the students needed to know in terms of content, but that it didn't affect how she taught because her teaching was adaptive to her students. Her main critique of the CS PD was that it became repetitive for teachers like herself who had been doing Zearn Math for a while. She suggested having more discussion questions or prompts with peers built in.

Classroom Observation

Juanita's classroom atmosphere was positive and vibrant, and students appeared eager to participate. There were no issues with student behavior. When students became overly animated, she easily reigned them in by clapping her hands, with students repeating back. She also anticipated potential issues—all the computers faced her so she could see if they were not doing Zearn Math (they all were).

Juanita's class worked on moving away from tape diagrams and area models and toward number lines in determining equivalent fractions. She had the class split into two sections: one worked on Zearn Math on the computers, and the other participated in the lesson. Halfway through the class, they switched. During the group lesson, she modeled a problem; then the students worked as a group to solve problems on the worksheet/board together, and then they worked independently on an exit ticket.

Mathematical discussions and student reasoning seemed common. For example, at one point during the observed lesson, Juanita made an error (on purpose) and asked students to correct her. She stubbornly debated students, which resulted in students using different techniques and building off of each other's responses seamlessly—all to prove her wrong. Students did not seem shy to explain their reasoning with each other or articulate their thinking to the teacher. While Juanita began the lesson with fill-in-the-blank questions, student reasoning took center stage as the lesson progressed, with Juanita shifting clarifying and elaborating on student contributions.

Teacher Perspectives on Instruction

Juanita said she tries to create an exciting environment where students feel comfortable and confident. When describing her instructional approach, she said she designs supports to be responsive to different types of learners and that differentiating was important to her. Unlike most of the other case study teachers, she highlighted the importance of the remediation

aspects in Zearn Math's curriculum. She said she appreciated that she could use Zearn Math to see where they need to build the foundational skills. She said she would give constant feedback via fist bumps, one-on-one discussions, and ask students to tell her if she's right or wrong. She also said she used a variety of strategies including cold calling on students, "turn and talks," "debate the teacher", and tried to make it playful so that students would have a positive relationship with math. She also seemed to have a thoughtful approach to drawing from the Zearn Math and Eureka curricula, both of which were used at her school. "When we first got the Zearn, I really liked the teacher lessons that they had on Zearn. I kind of meld those with the Eureka Math from Great Minds together, to create a balance of teacher-led discussion, because I love that part of the Zearn curriculum. The rigor, more upped difficulty that the Eureka Math has, so I kind of blend the two."

Juanita said that her first year of teaching was a struggle, particularly because she always hated math. Then, in her second year, when their school started using Zearn Math, she was able to use Zearn Math's videos and materials to see how to teach math effectively. This boosted her confidence in teaching math, and she said she had grown dramatically in her PCK since.

Jammie

Jammie taught fourth grade at Laurel and had been teaching for 12 years. She was one of two case study teachers with over 10 years of experience teaching.

CS PD Experience

Jammie was highly engaged with the CS PD and with her colleagues during the observed session, which was facilitated by the school's managing director for instruction in math. Teachers from both Laurel and Chabot participated. The facilitator made sure all the teachers shared their thoughts and ideas. She also pointed out things she observed Grade 5 students struggling with, for example drawing shapes proportionally so that they could make adequate comparisons of fractions, which allowed the Grade 4 teachers to see what skills to emphasize within the lesson. When working independently, teachers solved problems using Zearn Math methods (the area models they drew matched the video that played after their discussion). Jammie and other teachers in the CS PD session appeared to enjoy the fluency exercises and planned ways to integrate them into the class. The facilitator addressed responsive teaching moves or remediation. The teachers also brought up what they knew their third graders were good at from last year and how they could build on them.

In her instructional interview, Jammie said she appreciated that Zearn Math's CS PD showed different ways to teach a skill and solve problems in a visual manner and found the chunking of the videos to be helpful. Jammie's biggest challenge with the CS PD was its use of time.

Classroom Observation

Jammie's classroom was characterized by rules and behaviorist approaches to classroom management. The classroom atmosphere was generally positive. Students worked

collaboratively with each other. Jammie was strict and kept the class in constant motion to maintain student focus. For the most part, students were calm and quiet throughout the class. Jammie used a variety of classroom management strategies to keep students focused and engaged, such as awarding points to students for being attentive and prepared, using call and response, hand-clapping, calling out students who were talking out loud, or sending students to get a drink of water if they seemed to be nodding off. Students knew to raise their hands to request help and had different hand signals for things like going to the bathroom. They used several math phrases that the teacher reinforced through call and response. Students had math partners, whom they knew to work with when instructed to solve problems with a partner.

Jammie began the class by reviewing what students had learned in the previous lesson and had them work through a problem to illustrate the skills before introducing the current day's objectives. Jammie consistently switched between explaining the visual and explaining the symbolic arithmetic. Jammie would have students partition the area models, then explain what the new "like units" were and what each small square was "worth." Jammie would then show how she actually just multiplied the two fractions' denominators.

Jammie mostly asked "fill-in-the-blank" questions. She often restated student contributions and involved other students in more elaborate student-focused reasoning. When she worked through example problems, she frequently cold-called students to answer a specific part of the problem and then repeat answers for the class. All students were randomly called on to share responses to parts of example problems, and the environment was encouraging and collaborative with all students engaged to some degree. She encouraged students who she thought knew the answer to raise their hands. In one instance she projected students' worksheets on the board and talked through the different strategies they used to solve the problem. When providing individual feedback, Jammie largely drove the reasoning with some student input, by walking students through each problem-solving step rather than using prompts and questions to support students' own reasoning.

First, students solved example problems together as a full class, in pairs, and individually; then, all turned to individual work in the online Zearn Math component. During each problem that the class worked on, Jammie progressively provided less support. Students had choices about which strategy they used to solve problems in the teacher-led component and how quickly or how far they advanced through the online component.

Teacher Perspectives on Instruction

Jammie said focusing on the conceptual aspects of math were important, and that she used hand gestures and kinesthetic approaches as well as "acting things out." She also said aligning the Zearn Math missions and language with the state standards was important when using the Zearn Math curriculum. When she planned, she said she spent time looking over the big ideas from the Zearn Math lessons and then found ways to adapt the lesson for her students. She then aligned her objectives with the state math standards and created examples or exemplars.

She said she based her remediation on exit tickets. If students misunderstood concepts after a couple of days, she would pull a small group of students during the intervention block to re-teach. Jammie also said she aimed to create a positive and safe classroom environment for her students. "We cheer for each other. We clap. We work together as much as we can. We collaborate. I mean, we try to be very kind to each other as much as possible so they feel safe in answering. They know I'm going to call whoever I'm going to call. They know that it's not like a got-ya or something."

Jammie said Zearn Math helped her understand and appreciate that taking the time to build concepts slowly helps students in the long run. She said it made her realize it was worth spending more time on drawing models. However, according to Jammie, Zearn Math did not change the way she planned lessons.

Jammie said that Zearn Math influenced her pedagogical content knowledge by pushing her to teach using different strategies to cater to different types of learners. She also said that because Zearn Math prompts students to show their work in a way that makes their thinking visible and incorporates math discourse in lessons, she was able to understand how her students solve problems.

Demetris

Demetris taught fourth grade at Laurel and had an elementary education certificate. She had been teaching for three years. This was the first year she taught departmentalized math.

CS PD Experience

During the CS PD session, Demetris was engaged in discussing and collaborating with other teachers as well as solving problems authentically using Zearn Math strategies. She used multiple strategies to solve each problem independently and described different ways that she solved them during the discussion.

Demetris said that the overview provided at the beginning of CS PD sessions were always helpful because it gave teachers a recap of student skills and a preview of what she would be teaching students in the mission. She said that the CS PD sessions showed student thinking; however, she preferred the Zearn Math materials and curriculum over the PD session itself because the materials were more detailed.

Classroom Observation

Demetris's lesson was not observed due to COVID-19.

Teacher Perspectives on Instruction

Demetris said she prioritized ways to make math relatable to students, incorporating visuals, and differentiating groups. She said she tried to create a classroom culture that develops student independence and honesty. Her approach to whole class discussions was to have

students teach her the content, and then, as they walk her through steps if there are misunderstandings, have students help each other out. She said she differentiated how she engaged her students based on their characteristics. For those that were shy, she said she used popsicle sticks; and for those that were chatty, they got points. The school had scheduling structures that provided teachers time for remediation, which she said was helpful. She used this time to re-teach using her own materials or Zearn's materials. She said she provided multiple ways to provide feedback, including comments while grading, instilling the practice of students double-checking their work, and having a progress board tracking students' mastery for standards.

Demetris said she could see the concrete-pictorial-abstract learning process in action with the students. When they were first learning, they used pictorial diagrams; and then as they better grasped the concept, they dropped it. If Demetris's students were confused, she said she had observed them go back to the pictorial diagram.

Demetris said she felt very prepared to teach math and overall confident in the subject. She said that Zearn Math had allowed her to understand her students' thinking and given her the tools to teach students multiple strategies. She rated her PCK relatively strong, crediting Zearn Math and saying that on her own, her PCK would be much lower. She also said her knowledge of how students solve problems and, in particular, her knowledge of how students gain conceptual knowledge had grown because of Zearn Math.

Jard

Jard taught fourth grade and had been teaching for two years at San Lorenzo. Jard had an elementary certificate as well as a special education endorsement.

CS PD Experience

Jard came prepared to the CS PD session with a binder along with some supplemental materials. During the session, Jard was highly engaged in authentic problem-solving by working through the problems as if he were a student. At every video pause, he worked through each problem as a student would, by drawing a model, solving, and writing an explanation for his answer. He would even explain how to solve certain questions to others in the room and answered many questions about the Zearn Math methodology.

In the instructional interview, Jard said the CS PD sessions were useful because they set the teachers up for success. In the first year the school used Zearn Math (and Jard's first year of teaching), he said he did not take the CS PD sessions seriously. This year, Jard paid more attention and appreciated how Zearn Math helped him understand student thinking because it showed different ways students might work through problems.

Classroom Observation

Jard's classroom was characterized by several instances of student behavior that detracted from the lesson and required him to stop activities and to review behavioral norms. His instructional approach was primarily in the form of IRE, but he also built on student contributions. He frequently elicited student reasoning by following up on students' one-word answers to ask how they arrived at them. Many of his questions were also fill-in-the blank or factual. Responding to student contributions, he restated and evaluated. During the whole and small group lessons, Jard did most of the reasoning, though he sometimes asked students to explain the reasoning behind their answers and then built upon those answers to move the lesson along. He used the small group lessons as a way to remediate students with lower math knowledge. He strategically provided content for the students ahead of the whole-group review at the end of the class period and gave them the tools they needed to complete their online work. For small-group lessons, he started working with his lowest group and moved on from there, though he ran out of time to reach the higher students.

When students worked online, Jard encouraged them to use feedback from the online lesson instead of asking him. Students received stars for questions they answered correctly; and when they answered incorrectly, they were given additional chances to attempt their answer before the interface marked their solution as incorrect.

Teacher Perspectives on Instruction

Jard said he valued small group and one-on-one interactions with the students and that he tried to create a tight-knit family environment in his classroom. When asked in the instructional interview what ideas or principles were important to him when teaching Zearn Math, he said, "Follow the script because I have to." He seemed to struggle with the digital aspect of Zearn Math, often stating throughout the interview that he valued "hands-on" math and was struggling with adjusting to the new way of seeing students' thinking on the computer, as opposed to seeing it on paper. Jard also mentioned thinking that students spent too much time on the computer for Zearn Math.

It also seemed that Jard mostly viewed "Zearn" as the computer/digital aspect that students use, rather than the curriculum and materials. Jard said he valued the curriculum and CS PD sessions; however, when asked about Zearn Math as a whole changing his teaching practice, influencing his PCK, or changing how he viewed student thinking, Jard did not acknowledge the materials or curriculum, but rather only referenced the online component and stating that it did very little to influence or impact his teaching.

Diana

Diana taught fifth grade at San Lorenzo and had been teaching for 14 years. She was one of two teachers in the study with over 10 years of teaching experience. She had an elementary

education certification as well as a special education endorsement. This was Diana's first year teaching general education. Previously, she taught special education.

CS PD Experience

Diana was not very engaged during the CS PD—she would advance through sections very quickly, often not allowing time to solve the sample problems. She did engage a lot with the other teacher. In her instructional interview, she cited time being an issue and that she wished the PD was shorter which could explain why she moved through the session so quickly.

Diana liked that the PD gave her a preview of the mission. In terms of her classroom practices, she said that the CS PD helped prompt her to push her students to think rather than give them the answers. She did not think the PD helped her understand students' thinking or change the way she works with students because she said being around her students and getting to know them well was most helpful.

Classroom Observation

The classroom atmosphere in Diana's classroom was positive; students and the teacher were respectful of one another. Students responded well when she asked them to focus or quiet down. The observed lesson was a good example of conceptual exploration grounded in the concrete—she provided scenarios that students could imagine themselves in, involving students on a personal level. She also used visual tools, such as tape diagrams, to help students solve the lesson and problem set questions as she lectured. The lesson was coherent with the problem set and online work that students were asked to complete.

Diana asked students to create math sentences and to find "cues" in the problem to determine what operation they should use to solve it. She also asked students to create math sentences to express the problem. While these were more open-ended, or exploratory, most of her other questions were fill-in-the-blank questions or elicited yes or no answers. Diana mostly evaluated student contributions, although in the half-class lesson, she built more on student contributions, asking for cues and math sentences. Most of Diana's instruction was in the IRE format, but she also used and built on student responses to construct a collective understanding of how to approach the concrete problem she presented.

Teacher Perspectives on Instruction

When teaching math, Diana said in her instructional interview that it was important to anticipate student misunderstandings and to work through the problems in advance. She said she planned her math lessons by annotating the Zearn Math materials and pulling out key ideas to align with the exit ticket and assessments. She said she tried to create an open environment where students felt safe to take chances and discuss. She relied on the Zearn Math curriculum to bring discussion into her classroom as well as instances where she noticed misunderstandings.

Diana said that Zearn Math helped her understand math concepts at a deeper level. For example, instead of just doing an algorithm, she learned why the algorithm works. This in turn made her understand students' thinking better. Further, she said Zearn Math helped her teach to different learning styles. Diana said the digital aspect of Zearn Math incorporates remediation so that it is more customizable. She was now able to go back and see what concepts a student was struggling with and why. She said she relied on Zearn Math's digital program as remediation and did not elaborate on how she would help students who were not at the same level as their peers.

She also noted that the CS PD didn't particularly help with her preparation to teach math, but the collaboration that occurs during the PD helped her think differently about concepts.

Will

Will taught fifth grade at Fullerton and had an elementary education certificate. He was in his second year of teaching and had used Zearn Math for two years.

CS PD Experience

Will was the only case study teacher who completed the CS PD session alone and thus had no opportunity to collaborate. Throughout the session, he annotated the packet by circling or underlining the text alongside the video. During breaks in the video, he reviewed the sample question and worked through problems on paper. He used his own strategies, giving the impression of doing a quiz rather than authentically problem-solving.

In the instructional interview, Will said the CS PD sessions gave him insight into how he teaches and into student perspectives on the concepts and math problems. He said the CS PD helped him understand the importance of building on prior knowledge to understand the big ideas and concepts—even if some of the strategies were time-consuming.

Classroom Observation

Will created a positive, flexible classroom environment in which students were engaged in the content. There was some light teasing between teacher and students while they worked on the fraction conversions. When working through the problems, he made sure to adjust his speed and explanations to the level the students were at. For example, if they were stuck, he would slow down and address their misconceptions.

Will did most of the reasoning in the class. During the review of different strategies (repeated addition, standard algorithm, tape diagram, "the butterfly technique") he showed the students the steps for each of these strategies. While he didn't make it explicit to the students, this was one way in which they shifted from concrete to pictorial to abstract. After he showed the students how to solve the first problem using all the strategies, he encouraged students to find the way that worked best for them.

When providing feedback to students, Will mostly evaluated their work for correctness. His questions to students were primarily in the form of fill-in-the-blank. While students participated actively in the lesson, discussion or deep thinking about the math were less evident. Some concepts seemed rehearsed and call-and-response seemed to be a regular classroom practice. For example, students would chorally respond with the same answer when he asked certain questions. “What type of fraction is this?” “IMPROPER” students would say together. “Is there a remainder?” “NO!” “So is there a fraction?” “NO!” This appeared to engage students but did not lead to deeper discussions to push student thinking.

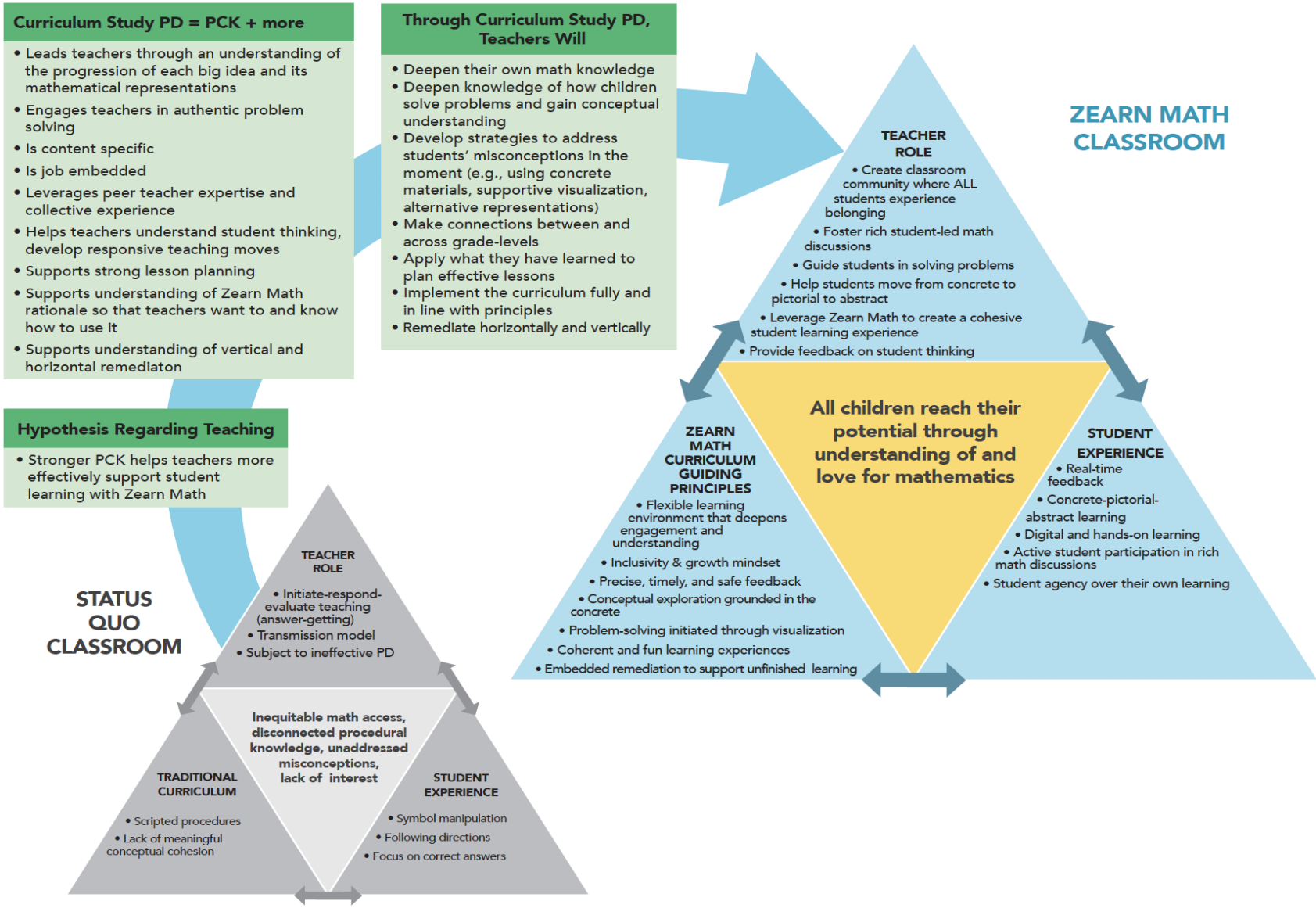
Teacher Perspectives on Instruction

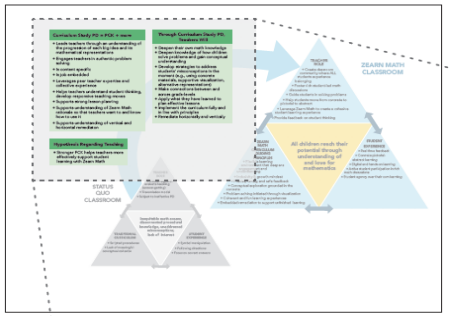
In the instructional interview, Will said the most important classroom practices were raising your hand, copying teacher notes, and not talking. The most important principles when using the Zearn Math curriculum, he said, were for students to be self-sufficient and to be able to use Zearn Math on their own without assistance from the teacher.

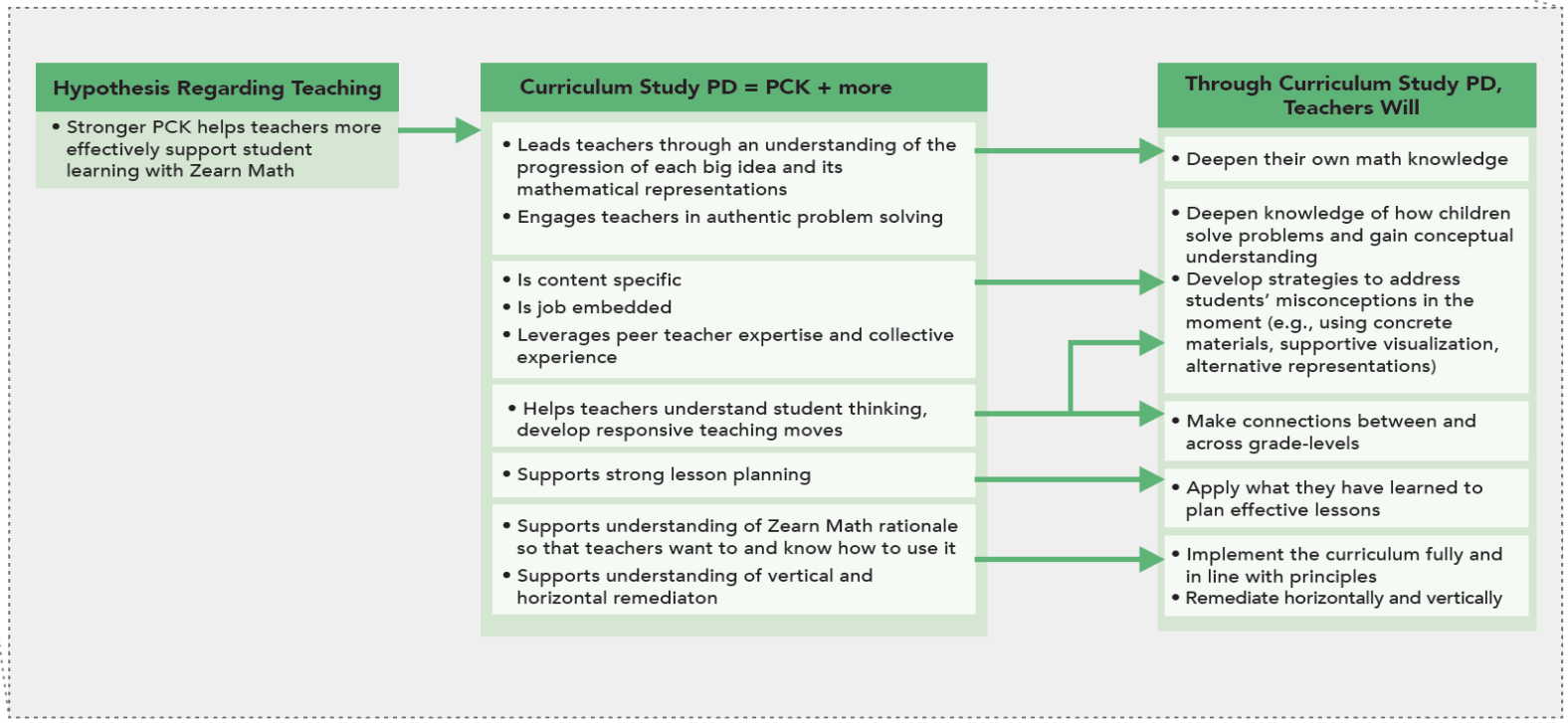
When lesson planning, Will said he looked at the lesson from previous years and adjusted and improved them based on his students’ past performance. In terms of remediation, he would make sure that he had multiple methods available for solving a problem. He said he would show students a different way of solving a problem if a student was unclear about a concept. When providing feedback on student work, he said he made sure to circulate the room and check to see if students were keeping up with him.

Will said he tried to ensure that his classroom was a free, safe space where it was acceptable to make mistakes and act like a kid. During classroom discussions, he said he made sure to promote deeper learning by asking “why” questions and prompting students to discuss with a partner to promote peer learning. To keep his whole class engaged, he said he used the cold call method to keep them on their toes and to prevent some students from dominating the discussion. Will said he thought students had agency because they were able to work as far ahead and as fast as they wanted. He said he liked that students were given multiple ways to solve problems and that they were able to choose the one that works best for them

Appendix B – Logic Model






Hypothesis Regarding Teaching
• Stronger PCK helps teachers more effectively support student learning with Zearn Math
Curriculum Study PD = PCK + more
• Leads teachers through an understanding of the progression of each big idea and its mathematical representations
• Engages teachers in authentic problem solving
• Is content specific
• Is job embedded
• Leverages peer teacher expertise and collective experience
• Helps teachers understand student thinking, develop responsive teaching moves
• Supports strong lesson planning
• Supports understanding of Zearn Math rationale so that teachers want to and know how to use it
• Supports understanding of vertical and horizontal remediaton
Through Curriculum Study PD, Teachers Will
• Deepen their own math knowledge
• Deepen knowledge of how children solve problems and gain conceptual understanding
• Develop strategies to address students' misconceptions in the moment (e.g., using concrete materials, supportive visualization, alternative representations)
• Make connections between and across grade-levels
• Apply what they have learned to plan effective lessons
• Implement the curriculum fully and in line with principles
• Remediate horizontally and vertically

Purpose of the Model

The Zearn Math logic model highlights the key aspects of Curriculum Study Professional Development (CS PD) and its intended outcomes on student learning and growth by making the following explicit:

- the problem CS PD is addressing (and its place in the teaching and learning system)
- a hypothesis about the roots of the problem
- the key aspects of the CS PD
- how CS PD figures in the overall teaching and learning system that leads to positive student outcomes

The purpose of the model is twofold: (1) to serve as a tool for Zearn to describe and explain the Curriculum Study model to teachers and other audiences; and (2) to inform SRI and TERC's study of CS PD. In terms of the latter, the logic model illustrates a shared understanding of what features are important for achieving desired teacher and student outcomes. SRI and TERC will use the logic model to focus the research—including the research questions, data collection instruments, and data analysis—on these features and desired outcomes.

Key Features of the Model

Problem (gray triangle)

The gray triangle illustrates three different aspects of the problem that the Zearn Math curriculum and Curriculum Study address. The triangle describes the status quo classroom with its traditional model of math instruction which includes the following:

- Teachers rely on initiate-respond-evaluate interactions with students (answer-getting); follow a transmission model of instruction, and are subject to ineffective PD.
- Traditional math curriculum often uses scripted procedures and lacks meaningful conceptual cohesion.
- Students do math mostly as symbol manipulation, and they are expected to follow teacher directions for how to manipulate symbols and produce correct answers.

The result (middle of the triangle) is that students have inequitable access to rich math learning experiences. Many simply learn procedures and formulas at the expense of deeper conceptual learning. This can lead to misconceptions about the nature of math that go unaddressed. These factors lead to many students losing interest and failing to develop a positive math identity.

Hypothesis Regarding Teaching (bottom green box)

As shown in the teal triangle, the teacher's role is critical to successful implementation of Zearn Math. Identifying a potential barrier to the successful implementation of Zearn Math, we and Zearn suggest the hypothesis that teachers need strong pedagogical content knowledge (PCK)

to move from stand-and-deliver teaching toward strong support of student learning with Zearn Math. As examples of this PCK in use, teachers are instrumental in ensuring that the principles and key features of Zearn Math are realized in ways that align with sound pedagogical practices, reflect an understanding of how students learn math, and respond directly to student needs.

Curriculum Study and What Teachers Will Be Able to Do (top left and right green boxes)

CS PD (top left green box) addresses PCK and other key aspects of teachers' roles by leading teachers through an understanding of the progression of each big idea in the curriculum and its mathematical representations and engaging teachers in authentic problem solving with those ideas. CS PD is content-specific and job-embedded, leveraging peer teacher expertise, and collective experience. It helps teachers understand student thinking and develop responsive teaching moves. It supports understanding of the Zearn Math rationale and strong lesson planning, as well as an understanding of vertical and horizontal remediation. As a result of participating in Zearn Math Curriculum Study PD, teachers gain the knowledge and strategies (top right green box) necessary to implement Zearn Math and support rich learning experiences for students.

Zearn Math Curriculum and Curriculum Study Outcomes (teal and yellow triangles)

Finally, the large triangle illustrates the Zearn Math Classroom: what math teaching and learning look like when the curriculum is implemented with fidelity, by teachers who have gained strong PCK through the Zearn Math Curriculum Study. The top teal triangle describes how teachers' instructional practices—supported by strong PCK—create an environment that fosters student growth and learning. These practices also ensure that teachers implement the Zearn Math curriculum aligned with its guiding principles (bottom left teal triangle). Students' experiences with teachers and curricular activities (bottom right teal triangle) are characterized by student agency, engagement in digital and hands-on activities, and the concrete-pictorial-abstract learning process that is at the core of the Zearn Math model. Teacher practices, the Zearn Math guiding principles, and students' experiences mutually reinforce each other and together support all students reaching their potential through understanding and love for math (middle yellow triangle).

Appendix C – PCK Interview Protocol

Thanks for being part of our study. We are trying to find out how Zearn Math curriculum study PD is working for teachers, and as part of that we want to learn more about how teachers think about their students' thinking. This is not a test of your teaching quality, nor a test of you personally at all. We will use the results to help us understand more about the PD. We'll want to ask you a similar question in the spring, after you have taught fractions.

I'm going to show you a fraction problem with a student answer. I'll give you some time just to think about it, out loud if you could. Then I will ask about how to help this child understand the concept the problem addresses. Just tell me what you think of, and I may ask questions along the way.

Share problem and student answer for the appropriate grade level with the teacher, on shared

video conference.

Ask teacher "What would you do to help this child understand the concept the problem addresses?"

Look for and prompt if needed:

1. What concept is being addressed
2. How to find out what the student is thinking
3. Multiple means for helping the child develop correct understanding of the concept.
4. Where did you learn that?

Appendix D – Teacher Instructional Interview Protocol

Zearn Math Teacher Interview Protocol

Consent

Provide teacher the consent form, ask them to read it through and ask any questions they have. Make sure they sign it. Do NOT start audio recording until they have verbally agreed to be recorded.

Is it okay If I audio-record the interview? Only the research team will hear the audio and see the transcription. We won't share what you said with your school or district, and when we report the data or use quotes, we make sure it's anonymous.

Introduction

Thanks again for taking the time to share your experiences with the Zearn Math Curriculum. As a reminder, the goal of this study is to evaluate the Zearn Math Curriculum and the Curriculum Study PD, not you or your students. The purpose of the interview is to learn first-hand from you what your experience has been so far, what works or doesn't work for you, and how Zearn can improve the curriculum and the PD.

Any questions before we start?

Background

1. How long have you been a teacher?
2. How long have you taught math?

Instructional Practices

3. * What classroom practices do you think are most important when you teach your students math?
4. * What ideas or principles are important to you when you use the Zearn Math curriculum?
5. * Describe how you plan your lessons.
    a. Has Zearn Math changed the way you plan your lessons?
    b. In what ways?
6. * Tell me in what ways you make connections between and across grade-levels when you talk to students about the math.
    a. Has Zearn Math helped you in this area at all?
    b. In what ways?
7. * Can you describe your approach to remediation?
    a. Have you learned anything different about how to approach remediation through the Zearn Math PD or lessons?
    b. Say more about that.

8. * Describe how you provide feedback to students on their work and progress.

Teacher Math Knowledge and Growth

9. * How well prepared do you feel to teach math?
10. * Has Zearn Math Curriculum Study helped you with this?
11. * How would you evaluate your own pedagogical content knowledge?
12. * In what ways, if any, has Zearn Math influenced your pedagogical content knowledge?
13. * Has your knowledge of how children solve problems changed?
    a. In what ways?
    b. Did Zearn Math influence that?
14. Has your knowledge of how students gain conceptual understanding changed?
    a. In what ways?
    b. Did Zearn Math influence that?
15. Have you learned any strategies for how to address students' misconceptions in their work or comments?
    a. If yes: Tell me more about what you have learned.
    b. If no: Do you feel like you have the strategies or tools you need to help students with misconceptions?
        i. What else do you need?

Curriculum Study PD Experience

Next, I have some questions about the Curriculum Study PD.

16. * What do you think was successful in the CS PD?
    a. What made it successful?
17. * What was challenging or did not work well in the CS PD?
    b. How come?
18. * How would you suggest improving the CS materials?
19. * How would you improve the CS PD sessions?
20. * Did the CS PD help you understand student thinking?
    c. If not, why not?
    d. If yes, in what ways?
21. * Did the CS PD help inform the ways in which you work with students?
    e. If not, why not?
    f. If yes, in what ways?
22. * Did the Curriculum Study PD connect with your own classroom practices?
    g. In what ways?

23. To what degree do you think Curriculum Study PD focused on relevant content?
24. In what ways did the CS PD introduce you to the progression of big ideas and math representations in the Zearn Math curriculum?
25. Tell me about your experiences of solving problems during CS PD.
    h. Did that experience help you with teaching the Zearn Math lessons?
26. Did you do the PD by yourself or with a colleague?
    i. If yes: Did the CS PD support collaboration with your colleague(s)?
        i. Was the collaboration beneficial?
        ii. In what ways?
    j. If no: Would any aspects of the PD have worked better if you did it with a colleague?
        i. In what ways?
        ii. Or did you do fine working on your own?

Learning Environment

27. How would you characterize the students in your classroom as a whole?
28. What kind of classroom culture do you try to create?

Student Experiences

29. * Tell me about how you approach discussions about the math in your classroom.
    a. How would you describe students' engagement in those discussions?
30. * To what degree do you think students have agency over their learning with the Zearn Math Curriculum?
31. One key aspect of the Zearn Math Curriculum is the concrete-pictorial-abstract learning process. Can you talk about what you have seen students doing with regard to moving through those three aspects of the math?
32. Do you see a good balance between digital and hands-on learning for students? Say more about that.

Closing

33. Is there anything else we haven't talked about you think would be important for us to know about your experience with the Zearn Math PD or curriculum?

Thank you!

Appendix E – Curriculum Study Observation Debrief Protocol

Questions about Curriculum Study PD		Debrief Notes
Summary of PD session	Describe the activities teachers participated in (just an overview of the basics)	
Teacher Engagement	Describe your impressions of how teachers engaged with the activities and each other	
What Worked	Describe your impressions of what seemed to work well	
Challenges	Describe your impressions of what did not work well, challenges teachers had with the activities or the computer, or anything else noteworthy	
Big Ideas	Did CS lead teachers through an understanding of the progression of each big idea and its mathematical representations?	
Authentic Problem Solving	Did CS engage teachers in authentic problem solving?	
Content	Was CS content specific? (The answer here will be yes, but describe anything noteworthy you observed about how teachers engaged with the math content)	
Job Embedded	Was CS job embedded (i.e., did it address teachers' own practices)?	
Collaboration	Did CS leverage peer teacher expertise and collective experience (did teachers collaborate)?	
Working with Student Thinking	Did CS help teachers understand student thinking and to develop responsive teaching moves?	
Lesson Planning	Did CS support strong lesson planning?	
Understanding Zearn Math Rationale	Did CS support understanding of Zearn Math rationale so that teachers want to and know how to use it?	
Supporting Remediation	Did CS support understanding of "vertical and horizontal remediation" (i.e., were teachers taught how to help students who are behind or struggling without holding them back a grade, keeping them at grade level)?	

Appendix F – Lesson Observation Debrief Protocol

	Notes
Teacher Pseudonym	
Date of observation	
Period	
Start time	
End time	
Minutes of instruction	
Observer	
Lesson	
Where class took place (classroom, lab)	
Comment on whether anything unusual or unexpected happened that disrupted the class in some significant way (fire drill, internet not working, etc.)	
Lesson Narrative. Provide narrative overview of the lesson that highlights the major events. This summary is intended to provide an overview of the lesson enactment for someone not present. Include the following elements in your summary: (a) if applicable, the stated lesson objectives, including posted, projected, or spoken agenda and/or standards; (b) the teacher's introduction to the observed lesson, including the "itinerary" and any verbal directions given by the teacher; (c) what you perceive as the learning goals of the lesson; (d) a description of the structure of the lesson and a brief note about the various activities; and (e) if available, information about the lesson context, that is, the lessons or activities that might have preceded or will follow the observed lesson.	
Classroom Management Describe classroom management and student behavioral issues	
Classroom Atmosphere Describe whether the atmosphere was positive, or if was there any teasing or negativity (between teachers and students and among students)	

	Notes
Classroom Math Norms: Describe any evidence of implicit classroom norms for: - mathematical discussion - help-seeking - collaboration - student independence (self-regulated learning, student reasoning)	
Fidelity to Zearn Math Principles Describe evidence of how the lesson reflected or contradicted the Zearn Math Curriculum Principles: - Flexible learning environment that deepens engagement and understanding - Inclusivity & growth mindset - Precise, timely, and safe feedback - Conceptual exploration grounded in the concrete - Problem-solving initiated through visualization - Coherent and fun learning experiences - Embedded remediation to support unfinished learning	
Student Feedback Describe the feedback you observed student receiving, from teacher and the digital interface	
Concrete-pictorial-abstract learning Describe instances of teacher and/or students focusing on or connecting concrete, pictorial, and abstract aspects of the math content.	
Digital & Hands-On Learning Describe the balance between student working digitally and hands-on	

	Notes
Teacher Questions Describe the kinds of questions teacher asks. Note which type of questions dominate. - Rhetorical (yes/no) - "Fill-in-the-blank" (teacher reasons, explains describes, and asks student questions to fill in, "as if" they are reasoning together) - Factual - Open-ended problem solving - Eliciting students' own reasoning - Eliciting examples - Clarifying student responses - Etc.	
Working with Student Contributions Describe how teacher responds to and builds on student contributions. Note which mode dominates. - Evaluates ("good," "nice," "incorrect," etc.). - Asks for clarification - Elaborates on - Restates - Builds on - Probes more extensively - Builds on and involves other students in more elaborate student-focused reasoning - Etc.	
Student Agency Describe who is doing the cognitive work during the lesson. - Teacher verbalizes her own reasoning - Teacher drives reasoning with some student input - Teacher and students collaborate to problem solve, explain, and reason - Students drive questions, conversation, reasoning	
Supporting Remediation Describe what remediation looked like, in terms of how teacher provided individual students help when they struggled.	
Mathematical Mistakes Describe any instances you saw of the teacher making mathematical mistakes (e.g. misrepresenting how to do an operation, using vocabulary incorrectly) and/or how s/he handled student mathematical mistakes (e.g. provided opportunity for learning or simply correcting students)	

	Notes
Overall Teaching Approach In summary, describe this teacher's teaching approach - IRE (Initiate, Respond, Evaluate) - Teacher-focused (teacher drives reasoning, lectures, assigns tasks, etc.) - Student focused (focus on student independence, initiative, ideas) - Adaptive - Direct instruction - Questioning - Project-focused - Personal and collaborative	

Appendix G – School Administrator Protocol

Zearn Math School Admin Interview Protocol

Consent

Email admin the consent form, ask them to read it through and ask any questions they have. Make sure they sign it, scan or take a photo of it with their phone and email it back. Do NOT start audio recording until they have verbally agreed to be recorded.

Is it okay If I audio-record the interview? Only the research team will hear the audio and see the transcription. We won't share what you said with your school or district, and when we report the data or use quotes, we make sure it's anonymous.

Introduction

Thanks again for taking the time to share your experiences with the Zearn Math Curriculum. As a reminder, the goal of this study is to evaluate the Zearn Math Curriculum and the Curriculum Study PD, not your school, teachers, or students. The purpose of the interview is to learn first-hand from you what your experience has been so far, what works or doesn't work for teachers and students, and how Zearn can improve the curriculum and the PD.

Any questions before we start?

Background

1. Tell me about your role at the school.
2. How long have you been there
3. What did you do before
4. Describe your role with regard to Zearn Math
5. How do you support teachers
6. What's your role with regard to Zearn Math Curriculum Study

Zearn in the School

7. Tell me about how Zearn Math was introduced to the school
8. How long has the school been using Zearn Math?

Teacher use of Zearn Math

9. Do teachers use other materials?
10. What benefits have you seen with Zearn Math?
11. What challenges

Curriculum Study PD

12. How has that been going
13. Facilitation?

14. What are you learning from teachers with regard to their experiences with Curriculum Study
15. Other math PD?
16. PLCs?
17. What do you think could be improved in the Curriculum Study materials or implementation?

Closing

18. Is there anything else we haven't talked about you think would be important for us to know about your experience with the Zearn Math PD or curriculum?

Thank you!

Appendix H – Teacher Background Survey

Zearn Math Study Teacher Background Survey

Start of Block: Block 1 - Consent

B1a

Welcome

Thank you for your participation in this research project, An Examination of Curriculum Study Professional Development through Multiple Lenses, that is being conducted on behalf of Zearn, by SRI International (SRI) and TERC.

What is the purpose of this project?

The purpose of the project is to gather information about how teachers experience and benefit from Zearn Math Curriculum Study Professional Development. Findings will be used to help Zearn modify and improve Curriculum Study.

The information we collect from you in this brief survey will be used to help us select a small number of teachers whom we will ask to participate in additional research activities to learn about how Curriculum Study is working for teachers. The information also helps us get a sense of how the selected case study teachers compare to peers in the school with regard to years of experience teaching, past experiences with professional development, and so on. Your responses will be confidential, and we will not share your information with your school, district, or anyone else outside the research group.

The survey should take 3-5 minutes of your time.

Who do I contact for more information?

If you have questions about this survey, please contact the Principal Investigator, Dr. Patrik Lundh, at 650-859-3603; email patrik.lundh@sri.com

Informed Consent

Please indicate below whether you have read and understood the information provided about this survey and that you agree to participate, or whether you decline to participate.

Consent

Please select one response:

- ○ I understand the statement above and I agree to participate in the study. (1)
- ○ I disagree to participate in the study. (2)

Display If Consent = 2

Thank you for your consideration.

Your survey has now been closed.

Close survey

End of Block: Block 1 - Consent

Start of Block: Block 2 - Survey

Name

What is your name?

First: (1) _____
Last: (2) _____

Email

What is your email address?

Email: (1) _____

District

What is your school district?

District: (1) _____

Page Break

Q1

1. How many years have you been teaching?

▼ 1 (1) ... 50 years or more (50)

Q2

2. How many years have you been teaching math?

▼ 1 (1) ... 50 years or more (50)

Q3

3. How many years have you been teaching grades 3-5?

▼ 1 (1) ... 50 years or more (50)

Page Break

Q4a

4. What teaching certificates do you hold?
(Select all that apply)

- ☐ Early Childhood (1)
- ☐ Early Childhood Special Education (2)
- ☐ Elementary Education (3)

Q4b
Secondary Education:

- ☐ English (4)
- ☐ Mathematics (5)
- ☐ Sciences - Biology (6)
- ☐ Sciences - Chemistry (7)
- ☐ Sciences - Life Sciences (8)
- ☐ Sciences - Physics (9)
- ☐ Social Sciences (10)

Q4c

- ☐ English as a Second Language (11)
- ☐ Special Education (12)
- ☐ Education Technology (13)
- ☐ Other (14)

Display If Q4c = 14

Q4_txt

Please specify other:

_____

_____

Page Break

Q5

5. What alternative teaching certificates do you hold?

_____

_____

_____

_____

_____

Q6

6. Approximately how many professional development offerings related to mathematics have you participated in during the past 5 years?

▼ None (0) ... 50 years or more (50)

End of Block: Block 2 - Survey

We thank you for your time spent taking this survey.

Your response has been recorded.

Appendix I – Constructs Table

Note that the constructs table was created based on the original research questions, which were consolidated and revised during the analysis process.

Research Questions	Sub-Questions	Constructs
1. How does the program, used within school-based professional learning communities, support teachers in developing and using pedagogical content knowledge to support student learning?	1.1. How do teachers characterize their status quo practice?	1.1.1. Teacher characterization of status quo work with students
		1.1.2. Teacher characterization of own status quo practices
		1.1.3. Teacher characterization of status quo classroom environment and culture
		1.1.4. Teacher characterization of own PCK
		1.1.5. Teacher characterization of status quo curriculum
	1.2. Do teachers have meaningful experiences with Curriculum Study as intended?	1.2.1. Content focus of Curriculum Study
		1.2.2. Curriculum Study connections with teacher practice (job embedded)
		1.2.3. Leading teachers meaningfully through progression of big ideas and math representations
		1.2.4. Engages teachers in authentic problem solving
		1.2.5. Help teachers understand student thinking, develop responsive teaching moves

Research Questions	Sub-Questions	Constructs
		1.2.6. Leverages peer teacher expertise and collective experience
	1.3. Does improved teacher PCK translate into improved teacher and classroom practices?	1.3.1. Teachers deepen their own math knowledge
		1.3.2. Teachers deepen knowledge of how children solve problems and gain conceptual understanding
		1.3.3. Teachers have strategies to address students' misconceptions in the moment (e.g., using concrete materials, supportive visualization, alternative representations)
		1.3.4. Teachers make connections between and across grade-levels
		1.3.5. Teachers apply what they have learned to plan effective lessons
		1.3.6. Teachers implement the curriculum fully and in line with principles
		1.3.7. Teachers remediate horizontally and vertically
	1.4. Do students have the intended experiences when teachers implement	1.4.1. Students receive real-time feedback
		1.4.2. Students engage in concrete-pictorial-abstract learning

Research Questions	Sub-Questions	Constructs
	the Zearn Math Curriculum and practices?	1.4.3. Students experience both digital and hands-on learning
		1.4.4. Students participate actively in rich math discussions
		1.4.5. Students have agency over their own learning
2. What factors contribute to success or failure of Curriculum Study within the context of professional learning communities?	2.1. What worked well in CS implementation?	2.1.1. Teacher's attributions of CS success
		2.1.2. Observations of what worked well in CS PD implementation
	2.2. What was challenging with CS PD implementation?	2.2.1. Teacher's attributions of CS failures or challenges
		2.2.2. Observations of what was challenging in CS PD implementation
	2.3. What patterns do we see in RQ1 data with regard to what worked well?	2.3.1. Analysis of RQ1 with focus on successes
	2.4. What patterns do we see in RQ1 data with regard to what was challenging?	2.4.1. Analysis of RQ1 with focus on challenges
3. How can Zearn improve its Curriculum Study?		3.1.1. Teacher feedback on CS materials

Research Questions	Sub-Questions	Constructs
	3.1. What additional needs to teachers report that CS does not meet?	3.1.2. Teacher feedback on CS PD format
		3.1.3. Teacher feedback on
	3.2. How do RQ2 findings inform improvement recommendations?	3.2.1. RQ2 analysis with focus on improvement recommendations
4. Do teachers gain pedagogical content knowledge after a year of Curriculum Study?	4.1. How do teachers' PCK change from the beginning to the end of the study?	4.1.1. Teachers' PCK in the beginning of the study
		4.1.2. Teachers' PCK at the end of the study
	4.2. How do teachers self-report on their growth in MKT?	4.2.1. Teachers' self-evaluation of MKT growth
	4.3. How do teachers self-report on their growth in PCK?	4.3.1. Teachers' self-evaluation of PCK growth

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